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# A semiparametric conditional capital asset pricing model ☆

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ABSTRACT

This paper proposes using a functional coefficient regression technique to estimate time-varying betas and alpha in the conditional capital asset pricing model (CAPM). Functional coefficient representation relaxes the strict assumptions regarding the structure of betas and alpha by combining the predictors into an index. Appropriate index variables are selected by applying the smoothly clipped absolute deviation penalty. In such a way, estimation and variable selection can be done simultaneously. Based on the empirical studies, the proposed model performs better than the alternatives in explaining asset returns and we find no strong evidence to reject the conditional CAPM.



## 1. Introduction

The capital asset pricing model (CAPM) plays a cornerstone role in theoretical and empirical finance. It states that a linear relationship exists between the excess return of a risky asset and the beta of that asset with respect to the market return. The betas in the CAPM are commonly assumed to be constant over time. However, recent empirical studies provide ample evidence against this assumption because the relative risk of firm's cash flow varies over the business cycle and the state of the economy; see, for example, Fama and French (1997), Ferson and Harvey (1997), Lettau and Ludvigson (2001), Zhang (2005), Lewellen and Nagel (2006) and the references therein. In other words, it is more reasonable to believe that the CAPM holds under the condition of current information sets, which leads to the conditional CAPM. In the conditional CAPM, the corresponding betas should be adjusted accordingly because information sets are updated over time. This implies that betas are time-varying. How to estimate the time-varying betas is of great importance because only when the betas are estimated appropriately, the pricing errors of the conditional CAPM are able to be measured correctly and the validity of the model can be evaluated. For this purpose, in this paper, we propose a new methodology to estimate the time-varying betas.

Estimation of the time-varying betas has already been discussed extensively through two different approaches in the finance literature. First, betas can be regarded as a function of time; for example, see the papers by Johnstone and Silverman (1997) and Robinson (1997). For parametric models in this approach, betas are assumed to be either a discrete function of time such as the threshold CAPM proposed by Akdeniz et al. (2003), or a continuous function like a smooth transition model developed by Lin and Teräsvirta (1994). For nonparametric models, betas are simply a nonparametric function of time, as described by Ang and Kristensen (2012) and Connor et al. (2012). This approach is criticized for hiding the economics driving force behind betas. It does not show how and why betas

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vary over time. The second approach is that betas are assumed to be affected by some variables. These variables can be the proxies of latent variables as pointed out by Ang and Chen (2007) or of some observable macro variables as in Ferson and Harvey (1999). It is clear that the latter approach can provide more economic intuition on the movement of betas than the former one. Because there is no overwhelming argument for either approach, this paper circumvents this debate and assumes that betas are functions of some observable variables. These variables are often called financial instruments.

There are two advantages of using financial instruments to track the movement of betas. The first advantage is that this method can reveal the close relationship between the relative risk of firm's cash flow as it varies over the business cycle and the state of the economy. As suggested by Aït-Sahalia and Brandt (2001), all of potential instrument variables can be combined into an index that best captures time variations in betas, and the index can be explained as an economic state variable. This index has a good interpretation as follows. From a statistical standpoint, the index avoids the curse of dimensionality because it allows us to reduce the multivariate problem to one. Therefore, we can implement the nonparametric approach (see Section 2 later) in a univariate setting; see Aït-Sahalia and Brandt (2001) for details. From an economic perspective, this index offers a convenient univariate summary statistic that describes the current state of the various time-varying economic indicators related to investment opportunities for portfolio investment. From a normative perspective, the index can help investors with any set of preferences to determine which economic variables they should track and, more importantly, in what single combination. The other advantage is that this approach considers not only the variation across averages of betas in each short time window but also the variation of the actual betas within each window. Indeed, Campbell and Vuolteenaho (2004), Fama and French (2005) and Lewellen and Nagel (2006), among others, assumed discrete changes in betas across sub-samples but constant betas within sub-samples.

However, there are still three pitfalls in this time-varying betas. First, there is a strict assumption about the relationship between the betas and the instrument variables. Ferson and Harvey (1999) imposed the assumption that the betas are linear functions of the index, whereas Wang (2002, 2003) found strong evidences against this assumption and argued that this strong assumption might lead to a model misspecification. As shown in Ghysels (1998), inference and estimation based on misspecification can be very misleading. In addition, Ghysels (1998) showed that among several well-known time-varying beta models, a serious misspecification might produce time variation in the beta that is highly volatile, and it might lead to large pricing errors. Thus, it is important to analyze the time-varying betas by relaxing the aforementioned assumptions. Second, as pointed out by Aït-Sahalia and Brandt (2001), it is often in the literature to choose instruments and to estimate model in two different model frameworks (under two different objective functions). In such a way, it might produce an inconsistent estimation or an inappropriate instrument selection. Therefore, to select instruments and best fit the model, the selection procedure should be conducted simultaneously with the estimation approach. Last, Harvey (2001) showed that the estimates of betas obtained using instrumental variables are very sensitive to the choice of instruments used as proxies for time-variation in the conditional betas.

To address the aforementioned issues, we propose using a functional coefficient regression (FCR) technique introduced by Cai et al. (2000) to estimate the time-varying betas and at the same time, adopting a penalty function to select the instrument variables. A FCR model estimates betas nonparametrically, by assuming that the coefficients of financial covariates are deterministic functions of some instrument variables. The estimates are obtained by using any nonparametric methods such as local linear fitting; see Fan and Gijbels (1996). Thus, a FCR model can relax the strong assumption of linearity and avoid model misspecification. The reader is referred to the survey paper by Cai and Hong (2009) on how to apply a FCR model in economics and finance. In addition, we can estimate betas and select instrument variables simultaneously by adding a penalty term. For example, we can choose the smoothly clipped absolute deviation penalty (SCAD) function introduced by Fan and Li (2001) although other penalty functions might be applicable. By doing so, the model estimation and variable selection can be implemented simultaneously so that important instrument variables are chosen automatically for the regression model. Thus, all of the potential candidates can be included into the model without examining whether a relationship exists between any individual instrument variable and the asset return. The main contribution of this paper is that a new FCR model with instrument variable selection is proposed from the conditional CAPM point of view. Also, the mathematical proofs for our model under time series settings are provided and indeed, they are different from the linear model set up in the paper by Fan and Li (2001) for the independent identically distributed (iid) sample. Moreover, the attractive point of this instruments selection for the FCR model is that it is not only critical for the conditional CAPM, but also flexible to be applied to other related economic and financial areas where the important variables should be selected from very large scale candidates.

In this paper, a discussion concerning about time-varying betas is under the framework of the conditional CAPM. However, the conditional CAPM, as an alternative to the static CAPM, is quite controversial in the finance literature; see Lewellen and Nagel (2006) for the detailed arguments. This means that it is difficult to choose the conditional CAPM or the static CAPM in a real application. Theoretically, the conditional CAPM can hold perfectly from period to period, and it can be regarded as a base for many other models; for example, the premium-labor model in Jagannathan and Wang (1996). Conversely, Ferson and Harvey (1999), Wang (2003) and Lewellen and Nagel (2006) found that the conditional CAPM is rejected by their empirical analysis, although their results might not be convincing. As for the aforementioned papers, the model in Ferson and Harvey (1999) may be misspecified; see Section 4 later for the detailed arguments. Wang (2003) used four instrument variables to perform nonparametric estimations and tests. However, the number of observations in his data might not be large enough to produce reliable inferences. Finally, Lewellen and Nagel (2006) did a simple comparison to evaluate whether the variation in the betas and the equity premium is large enough to explain important asset-pricing anomalies. However, the size of their test method is challenged by Li et al. (2015). Thus, the validity of choosing the conditional CAPM over the static CAPM is still an open research topic.

Recently, Ferreira et al. (2011) proposed a nonparametric two-stage estimator for conditional beta pricing models by allowing for flexibility not only in the betas but also in the risk premium. Our method can be considered as a generalization of the first stage estimation in Ferreira et al. (2011) with the advantage that there is no need to select the instrumental variables in advance. In addition, Ferreira et al. (2011) used multivariate kernel to estimate the beta, and their method suffers from the curse of dimensionality. For this reason, in their empirical analysis, they only used two instrument variables. While, our model can alleviate this problem in the estimation. Connor et al. (2012) developed a characteristic-based weighted additive regression for the factor model. Due to the curse of dimensionality, univariate nonparametric functions are considered as the characteristic-betas in the paper. In their empirical analysis, four characteristics including size, value, momentum