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Modeling interest rate volatility: A Realized GARCH approach

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ABSTRACT

We propose using a Realized GARCH (RGARCH) model to estimate the daily volatility of the short-term interest rate in the euro–yen market. The model better fits the data and provides more accurate volatility forecasts by extracting additional information from realized measures. In addition, we propose using the ARMA–Realized GARCH (ARMA–RGARCH) model to capture the volatility clustering and the mean reversion effects of interest rate behavior. We find the ARMA–RGARCH model fits the data better than the simple RGARCH model does, but it does not provide superior volatility forecasts.

1. Introduction

Short-term interest rates are widely recognized as key economic variables. They are used frequently in financial econometrics models because they play an important role in evaluating almost all securities and macroeconomic variables. However, many popular models fail to capture the key features of interest rates and do not fit the data well. A milestone in terms of interest rate models was the development of the generalized regime-switching (GRS) model, proposed by Gray (1996). Conventional GARCH-type and diffusion models failed to handle certain interest rate events, such as explosive volatility, which would cause serious problems in certain applications. He believed this failure may be due to time variations in the parameter values. The GRS model nests many interest rate models as special cases, and allows the parameter values to vary with regime changes. Thus, the model should provide a solution to the problem. He found that the GRS model outperforms conventional single-regime GARCH-type models in out-of-sample forecasting. Unfortunately, the GRS model still does not fit the data well, since almost all the reported parameters of the most generalized version are nonsignificant.

Another conceivable reason for the failure to model short-term interest rates adequately is that the σ-field is not sufficiently informative. When modeling interest rates using conventional GARCH-type models, the only data used are the daily closing prices. All data during trading hours are ignored. As high-frequency data have become more available, recent literature has introduced a number of more efficient nonparametric estimators of integrated volatility (see Barndorff-Nielsen and Shephard, 2004; Barndorff-Nielsen et al., 2008; and Hansen and Horel, 2009). Examples of models that incorporate these realized measures include the multiplicative error model (MEM) of Engle and Gallo (2006), and the HEAVY model of Shephard and Sheppard (2010). These models incorporate multiple latent variables of daily volatility. Then, within the framework of stochastic volatility models, Takahashi et al. (2009) propose a joint model for return and realized measures. Shirota et al. (2014) introduce the realized stochastic volatility (RSV) model, which incorporates leverage and long memory. A Heston model studies the joint behavior of return and volatility, and shows decisively different extremal behavior to the conventional GARCH and SV models proposed by Ehlert et al. (2015).

In this study, we model the short-term interest rate in the euro–yen market using the Realized GARCH (RGARCH) framework proposed by Hansen et al. (2012). Section 2 introduces the RGARCH framework, including the log-linear specification, the estimation method, the conditional distributions, the robust QLIKE loss function, and the value-at-risk-based loss function. Here, we also

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present the model confidence set (MCS) procedure we use to evaluate the volatility forecasting performance. Section 3 provides a brief description of our data and the realized measures. Section 4 reports the in-sample empirical results and evaluates the rolling-window volatility forecasting performance. Then, in Section 5, we introduce an ARMA–Realized GARCH (ARMA–RGARCH) framework. Here, we present our empirical results, and compare them to those of the simple RGARCH model. Lastly, Section 6 concludes the paper.

2. Methodology

2.1. Model specification

We adopt the RGARCH model with its log-linear specification, as proposed by Hansen et al. (2012). In Section 5, we propose an extension of the model to capture the well-known mean reversion and volatility effects in short-term interest rate behavior. The RGARCH(p,q) model is specified as follows:

$$r_t = \sqrt{h_t} z_t \tag{1}$$

$$\log h_t = \omega + \sum_{i=1}^{p} \beta_i \log h_{t-i} + \sum_{j=1}^{q} \gamma_j \log x_{t-j} \tag{2}$$

$$\log x_t = \xi + \varphi \log h_t + \tau(z_t) + u_t, \tag{3}$$

where $z_t \sim I.I.D(0,1)$, $u_t \sim I.I.D(0,\sigma_u^2)$, r_t is a zero-mean series, and h_t and x_t denote the conditional variance and realized measure, respectively. Then, $\tau(z_t)$ is the leverage function constructed from Hermite polynomials. Here, we adopt the simple quadratic form: $\tau(z_t) = \eta_1 z_t + \eta_2(z_t^2 - 1)$. This choice is proper for two reasons. First, it satisfies $E[\tau(z_t)] = 0$ for any standard distribution with $E[z_t] = 0$ and $Var[z_t] = 1$. Second, it is proportional to the news impact curve discussed in Engle and Ng (1993). Thus, it can capture the asymmetric effect of price shocks on daily volatility. The news impact curve is defined as $v(z) = E(\log h_t | z_{t-1} = z) - E(\log h_t)$.

Eqs. (1) and (2) construct a GARCH-X model, with the restriction that the coefficient of the squared return is zero. Eq. (3) is called the measurement equation, because x_t is a measure of h_t. The measurement equation completes the model, using the leverage function $\tau(z_t)$ to provide a simple way to investigate the joint dependence of r_t and x_t. Numerous studies on market microstructures argue that returns are dependent on trading intensity and liquidity indicators, such as volume, order flow, and the bid–ask spread (see Amihud (2002), Admati and Pfleiderer (1988), Brennan and Subrahmanyam (1996), and Hasbrouck and Seppi (2001)). Because the intraday volatility is linked to the volume, order flow, and trading intensity directly, it should be dependent on the return. Therefore, the measurement equation is important from a theoretical point of view. Unlike other models that incorporate realized measures in the variance equation, such as the MEM and HEAVY models, the realized measure is not treated as an exogenous variable. When the realized measure is a consistent estimator of integrated volatility, it should be viewed as the conditional variance plus an innovation term. The conditional variance is adapted to a much richer σ-field, $F_t = \sigma(r_t, r_{t-1}, \ldots, x_t, x_{t-1}, \ldots)$. Conventional GARCH and SV-type models only use daily closing prices. In comparison, the additional information included in the realized measure on intraday volatility is expected to promote the fit to the data and the forecasting accuracy of conditional volatility. Moreover, the logarithmic conditional variance can be shown to follow an ARMA process:

$$\log h_t = \omega + \sum_{i=1}^{p\vee q} (\beta_i + \varphi\gamma_i) \log h_{t-i} + \sum_{j=1}^{q} \gamma_j [\tau(z_{t-j}) + u_{t-j} + \xi]. \tag{4}$$

The logarithmic conditional variance, h_t, is driven by both the innovation of return and the realized measure. The leverage effect is already indirectly embedded in the variance equation. The persistence parameter, π, is given by $\pi = \sum_i (\beta_i + \varphi\gamma_i)$.

2.2. Estimation method

Following Hansen et al. (2012), we summarize the estimation method in this section. The model is estimated using the quasi-maximum likelihood (QML) method. The likelihood function of the Gaussian specification is given by

$$l(r,x;\theta) = -\frac{1}{2}\sum_{t=1}^{n}\left[\log h_t + \frac{r_t^2}{h_t} + \log(\sigma_u^2) + \frac{u_t^2}{\sigma_u^2}\right]. \tag{5}$$

Since conventional GARCH-type models do not model realized measures, it is meaningless to compare the joint log likelihood to those of the conventional GARCH models. However, we can derive the partial likelihood of the RGARCH model and compare that with the log likelihood of the GARCH models. The joint conditional density of the return and realized measures is

$$f(r_t, x_t | F_{t-1}) = f(r_t | F_{t-1})\, f(x_t | r_t, F_{t-1}). \tag{6}$$

Then, the logarithmic form is

$$\log f(r_t, x_t | F_{t-1}) = \log f(r_t | F_{t-1}) + \log f(x_t | r_t, F_{t-1}). \tag{7}$$

Thus, the joint log likelihood of z_t and u_t under the Gaussian specification can be split as follows:

$$l(r,x;\theta) = -\frac{1}{2}\sum_{t=1}^{n}\left[\log(2\pi) + \log h_t + \frac{r_t^2}{h_t}\right] - \frac{1}{2}\sum_{t=1}^{n}\left[\log(2\pi) + \log(\sigma_u^2) + \frac{u_t^2}{\sigma_u^2}\right]. \tag{8}$$

Then, the partial likelihood of the model is defined as

$$l(r;\theta) = -\frac{1}{2}\sum_{t=1}^{n}\left[\log(2\pi) + \log h_t + \frac{r_t^2}{h_t}\right]. \tag{9}$$

Hansen et al. (2012) obtained that $\sqrt{n}(\hat{\theta}_n - \theta) \to N\left(0, I_\theta^{-1} J_\theta I_\theta^{-1}\right)$ where expressions for I_θ and J_θ are given in the Appendix.

An alternative estimation method for GARCH-type models is proposed by Ossandon and Bahamonde (2011). It is possible to have a novel state space representation and an efficient approach based on the Extended Kalman Filter (EKF). Since the structure of the RGARCH model is quite complicated, this is left as a topic for further research.

We also change the assumption of a standard normal distribution on z_t to a more realistic distribution. Here, we adopt the distribution proposed by Fernandez and Steel (1998) that allows for skewness in any symmetric and continuous distribution by changing the scale of the density function:

$$f(z|\varsigma) = \frac{2}{\varsigma + \varsigma^{-1}}[f(\varsigma z)H(-z) + f(\varsigma^{-1} z)H(z)], \tag{10}$$

where ς is the shape parameter and $H(.)$ is the Heaviside function. The distribution is symmetric when ς is equal to 1. The mean and variance are defined as

$$\mu_z = M_1(\varsigma - \varsigma^{-1}) \tag{11}$$

$$\sigma_z^2 = (M_2 - M_1^2)(\varsigma^2 + \varsigma^{-2}) + 2M_1^2 - M_2, \tag{12}$$

respectively, where

$$M_k = 2\int_0^\infty z^k f(z)dz. \tag{13}$$