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# Quote inefficiency in options markets

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ABSTRACT

In an arbitrage-free economy with non-zero bid-ask spreads the existence of payoffs whose price is lower than the price of a dominated payoff cannot be discarded in general. However, when the former price corresponds to trivial portfolios which involve buying or selling one unit of the basis assets, its presence, although not an arbitrage, is a severe market anomaly which we refer to as an inefficient quote. In an empirical study, we report evidence that indicates that in options markets both the frequency and the magnitude of these anomalies are substantial and we document puzzling patterns in their behavior.


## 1. Introduction

Simple two-period models have provided a very useful tool for the pricing of European options and futures contracts. In addition, many of the advances have been accomplished by resorting to fairly weak economic assumptions, namely, the law of one price, absence of arbitrage and absence of second-order stochastic dominance.

The first one and weakest of these assumptions simply precludes different prices for identical payoffs. Most tests of the law of one price are designed to check the validity of one of its specific pricing predictions. For example, Gould and Galai (1974), Klemkosky and Resnick (1980, 1979) and Kamara and Miller (1995), to mention just a few, concentrate their efforts on the well-known put-call parity. Protopapadakis and Stoll (1983) and Yadav and Pope (1994) empirically examine the relationship between spot and futures prices whereas Bharadwaj and Wiggins (2001) look into potential mispricings of Box Spread combinations.

Absence of arbitrage is a slightly stronger departure point, which thus includes the law of one price as a special case. In a frictionless market it can be linked to the existence of a bounded solution to the two-period portfolio problem for at least one investor with strictly monotonic preferences, or equivalently, to the existence of a strictly positive stochastic discount factor (SDF). The additional pricing constraints that this assumption brings about are unfortunately less tight and must be formulated in terms of price bounds. Work in this direction was pioneered by Perrakis and Ryan (1984) and Ritchken (1985). Tests of no-arbitrage conditions in this simple framework can be found in Balbás et al. (2000), Balbás et al. (1999) and Ackert and Tian (2001).

A third turn of the screw can be accomplished by resorting to the absence of second-order stochastic dominance. The focus is now in discarding a price which will prevent any investor with monotonic and concave preferences from taking a position in the corresponding asset. This approach has been proved fruitful and it has managed to deliver bounds which are tighter than their no-arbitrage counterparts. Important contributions in this area are Levy's (1985) seminal work and more recently Constantinides and Perrakis (2002). On the empirical front, Constantinides et al. (2009) under quite general assumptions report widespread instances of second-order stochastic dominance.

Recent developments in option pricing seem to indicate that the above assumptions have done their job and that all pricing implications based on them have been exhausted. This is the implicit conclusion that one can draw from, for example, the price bounds

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**Table 1**
This table displays a subset of best bid and ask quotes that were observed on October 10, 2006 at 13:36 for EOM European options on the CME E-mini S&P 500 Future Index with maturity November 29, 2006. The first two columns indicate the asset, the third and fourth columns show the bid and ask prices and their associated volumes are displayed in columns six and seven. The strike of the options is listed in column eight. The current price of the Index is 1355.5. All prices are expressed in index points. The bond quotes refer to the price of one index point to be delivered at maturity. The last two columns present the optimal super replicating portfolio of the inefficient bid quote of asset 4.

| Asset | Type | Bid | Ask | Vol. bid | Vol. ask | Strike | $w^b$ | $w^a$ |
|---|---|---|---|---|---|---|---|---|
| 1 | Bond | 0.992869 | 0.992869 | – | – | – | 150 | 0 |
| 2 | Call | 18.50 | 18.75 | 81 | 20 | 1360 | 0 | 1 |
| 3 | Put | 23.25 | 24.00 | 61 | 213 | 1360 | 1 | 0 |
| 4 | Put | 150.75 | 155.75 | 60 | 60 | 1510 | 0 | 0 |

derived by Cochrane and Saa-Requejo (2001), and Bernardo and Ledoit (2000).

As it turns out, we can go one step backward rather than forward in our list of assumptions and find, as our arguments will show, unexploited meaningful implications for European options and futures prices which do not even require the concavity assumption on preferences associated to the lack of second-order stochastic dominance.

The focus of analysis in this paper may be illustrated with a real life example. Table 1 reports a subset of best bid and ask quotes that were observed on October 10, 2006 at 13:36 for End-Of-the-Month (EOM) European options on the CME E-mini S&P 500 Futures Index with maturity November 29, 2006. Prices are expressed in index points and the current price of the Index is 1355.5.

Arbitrage is not possible at these prices; however, one can still wonder if it is possible to superreplicate the payoff resulting from buying (selling) any of this individual options at a price that is lower (higher) than their associated best ask (bid) quote by using a combination of the available assets.[1] As it turns out, examining this possibility is remarkably simple.

Consider, for example, a short position on Asset 4 which at this point would give its holder 150.75 index points and whose payoff is

$$-\max(1510 - S_T, 0) \tag{1}$$

where $S_T$ denotes the uncertain value of the underlying index at maturity. Any alternative portfolio of the available assets would now reward its owner with an amount equal to

$$0.993w_1^b - 0.993w_1^a + 18.5w_2^b - 18.75w_2^a + 23.25w_3^b - 24w_3^a + 150.75w_4^b - 155.75w_4^a \tag{2}$$

and it will deliver a payoff at maturity given by

$$w_1^a - w_1^b + (w_2^a - w_2^b)\max(S_T - 1360) + (w_3^a - w_3^b)\max(1360 - S_T) + (w_4^a - w_4^b)\max(1510 - S_T, 0). \tag{3}$$

where $w_i^a$ and $w_i^b$ denote the nonnegative weights of the long and short positions on asset $i$, respectively.

The goal is thus to determine whether there exist weights for which the maximum value of (2) exceeds 150.75 and whose associated payoff (3) is greater or equal than (1) for all possible values of $S_T$. This latter condition may be found difficult to examine since it involves in principle an infinite number of constraints. Fortunately, almost all of them are redundant. First, note that although $S_T$ can take any nonnegative value, it is fairly safe to expect that it will never exceed, for example, 10 times the current value of the underlying, so that it can be assumed to lie in the interval $(0, 13555)$. Second, the payoff of the portfolio is a piecewise linear function of $S_T$ with kinks at the strike values of the options included. In this particular example, this payoff will at most have kinks at 1360 and 1510. As a result, it is easy to see that (3) will be greater than (1) for all values of $S_T$ in $(0, 13555)$ if and only if it satisfies such condition for all values of $S_T$ in $\{0, 1360, 1510, 13555\}$. The feasible set of super-replicating portfolios is thus defined by these five inequalities together with the nonnegativity constraints on the portfolio weights. Hence, our task of maximizing (2) subject to the super-replicating condition is reduced to solving a simple linear program with eight decision variables and twelve constraints. In our example, the optimal value of its objective function is equal to 153.43 and the corresponding vector of optimal weights is given in Table 1. A graphical description of the payoffs involved is presented in Fig. 1.[2]

Clearly, this situation is at odds with a competitive price setting process for two reasons. Firstly, no trader will ever accept the best bid quote for asset 4 (at least a rational investor who is not exposed to exorbitant transaction fees) since the alternative portfolio is clearly superior. This renders the bid quote as uninformative. Secondly, any market maker can offer a better bid price within the interval $(150.750, 153.43)$, for example, 152.If this position is taken and therefore she is required to purchase the inefficient put, she only needs to implement the super-replicating portfolio. Her combined position will give a payoff at maturity identical to the one resulting from buying a call option with strike price 1510, a payoff which is obviously greater or equal than zero regardless of what the final value of the underlying is. Thus, she can pocket the difference $153.43 - 152$ without bearing any risk at all. Furthermore, if the underlying at maturity happens to be above 1510, an additional profit of $S_T - 1510$ index points may be obtained. This is not an arbitrage opportunity but it is indeed the opportunity of an arbitrage opportunity.

In formalizing this new concept, our theoretical and empirical considerations accommodate the presence of volume constraints and trading fees. Furthermore, in order to go beyond an analysis of the efficiency of individual quotes, we introduce a measure that quantifies the overall degree of quote inefficiency of the market. Our analysis has the virtue of simplicity since, as it has been illustrated above, the definition of these objects and its computation involves simple linear programming.

With these theoretical tools in our hands, we empirically examine the quality of the price-setting process of options market participants. Our evidence indicates a clear presence of these anomalies for a large sample of EOM European options on CME E-mini S&P 500 futures. Furthermore, a deeper analysis of our results shows some puzzling patterns in the behavior of these mispricings.[3] Specifically, we find that quotes tend to be on average inefficient when the associated option, be it a call or a put, is in the money, whereas they are highly efficient when the options lie

[1] At this minute there were quotes available for the future contract and a much larger number of options. However, for the sake of simplicity and for illustrative purposes, we do not consider them.

[2] In order to improve the visibility of the graph, we only plot the payoffs for values of the underlying up to 5000 instead of 13555.

[3] The term mispricing is perhaps abusive in as much as these inefficiencies are not outright violations but rather opportunities available to sophisticated traders to achieve certain payoffs via a cheaper superreplicating strategy. However, as exemplified here, they also open the possibility to implement an arbitrage strategy whenever an inefficient quote is accepted.