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Credit spreads and state-dependent volatility: Theory and empirical evidence ☆

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ABSTRACT

We generalize the asset dynamics assumptions of Leland (1994b) and Leland and Toft (1996) to a state dependent variance with constant elasticity process (CEV) and obtain analytical solutions for corporate debt and equity value. We use the GMM technique to extract the parameters by fitting the empirical data in the equity and credit default swap markets simultaneously. We find that the elasticity parameter is significantly different from zero for most of the firms and that the CEV model performs much better than the model with constant volatility in both in-sample fittings and out-of-sample predictions of CDS spreads.

1. Introduction

A very large number of studies, both theoretical and empirical, on corporate bond pricing and the risk structure of interest rates have appeared in the literature following the pioneering work of Merton (1974) and Black and Cox (1976), which in turn were inspired by the seminal Black and Scholes (1973) model of option pricing. These studies adopted the methodological approach of contingent claims valuation in continuous time, in which the value of a firm's assets played the role of the claim's underlying asset and allowed the valuation of the various components of the balance sheet under a variety of assumptions. This approach has been shown to be sufficiently flexible to tackle some of the most important problems in corporate finance, such as capital structure, bond valuation and default risk, under a variety of assumptions about the type of bonds included in the firm's liabilities. The resulting models came to be known as *structural models* of bond pricing, as distinct from another class of models known as reduced form models, in which there is no direct link between the bonds of a given risk class and the firm's capital structure.[2]

Under continuous coupon payment and first-passage default[3] assumptions, Leland (1994a,b) and Leland and Toft (1996) first studied corporate debt valuation and optimal capital structure with endogenous default boundary for infinite and finite maturity debt, respectively.[4]

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[2] For the reduced form models see Jarrow and Turnbull (1995) and Duffie and Singleton (1999). These models lie outside the topic of this paper.

[3] Under the first-passage default assumption, a firm will claim default when the asset value first crosses the pre-determined default boundary. This default boundary can be determined endogenously (Leland, 1994a,b; Leland and Toft, 1996; Duffie and Lando, 2001) or exogenously (Longstaff and Schwartz, 1995).

[4] Leland (L, 1994a,b) and LT use the asset value of the unlevered firm as the basic underlying process for the valuation of the various components of the balance sheet of the levered firm. In a variant of the basic model, presented in Goldstein et al., 2001, the firm value is estimated from the dynamics of the earnings before interest and taxes (EBIT), split between the claimholders and the government; see also Sarkar and Zapatero (2003).

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Because of the computational complexity of the valuation expressions a major emphasis in the structural models was placed on the derivation of closed form expressions, rather than numerical results based on approximations[5] or simulations.[6] Such a focus allowed relatively easy estimations of numerical values given the parameters of the model, but at the cost of maintaining simple formulations of the mathematical structure of the asset value dynamics, in which a univariate diffusion process still follows the original Black and Scholes (1973) and Merton (1974) assumption of a lognormal diffusion with constant volatility.[7] This is all the more surprising, in view of the fact that the option pricing literature has long recognized that such an assumption is no longer adequate to represent underlying assets in option markets, and has introduced factors such as rare events, stochastic volatility and transaction costs. Choi and Richardson (2009) studied the conditional volatility of the firm's asset by a weighted average of equity, bond and loan prices and found that asset volatility is time varying. Similarly, Huang and Zhou (2008), in their study of the term structure of credit default swaps (CDS), note that time varying asset volatility should potentially play a role in structural models in order to fit into the empirical credit default spread.

In this paper we generalize the dynamics of the asset value by assuming that the diffusion volatility is state-dependent, varying with the asset value. We use a particular form of volatility state dependence, known as the Constant Elasticity of Variance (CEV) model, originally formulated by Cox (1975) in the context of option pricing.[8] Compared to constant volatility diffusion, the CEV model has only one extra parameter, the elasticity of variance, and includes constant volatility as a special case. Although this extension introduces significant additional computational complexity, we manage to derive closed form expressions for almost all the variables of interest, including corporate debt value, total levered firm value, optimal leverage and equity value. Both Leland (1994b) and LT are special cases of the CEV model with zero elasticity of variance. The results presented in the body of the paper refer to the L model, while the equivalent results for LT are presented in the appendix.

Our numerical simulations show that the elasticity parameter plays a major role in the determination of the endogenous default boundary of the L and LT models, with a negative (positive) parameter decreasing (increasing) the boundary compared to the constant volatility case. These relative sizes of the default boundary hold for all maturities and all leverage ratios. The elasticity parameter also has a strong impact on credit spreads, with negative (positive) values widening (narrowing) the spread for all maturities in comparison with the constant volatility case, especially for exogenously set default boundary; this effect still persists but in a weaker form when the boundary is endogenous. Similar elasticity parameter effects are present in the determination of the optimal leverage and the volatilities of the equity and debt. As in Huang and Huang (HH, 2012), we also estimate the empirical default probability, the equity premium and the leverage ratio and find that the empirically documented negative elasticity parameter boosts the percentage of yield spread due to default significantly, especially for debt with longer maturity and lower ratings.

The main empirical results of this paper, however, consist of a comparison of the performance of the two alternative volatility assumptions (constant and CEV) within the context of the debt assumptions of the Leland (1994b) model. Using time series data from equity and CDS market for a sample of firms, we estimate the parameters using the Generalized Method of Moments (GMM) method by fitting the two competing models to all the available historical data, including both the implied equity volatilities and the credit default swap (CDS) data. We document that the CEV structural model with an elasticity parameter of around -0.54 on average exhibits a superior fitting in the CDS spreads across all maturities. The relationship between the sign and value of the elasticity parameter and the firm specific measures of default risk, such as leverage ratios, CDS spreads and current ratios indicates that there is a tendency for β to increase as the risk of the firm decreases, but that the tendency is weak and fluctuates.

We confirm the superiority of the CEV model by systematic comparisons of the goodness of fit of the CDS data for each firm and each maturity, both in- and out-of-sample. We find that the constant volatility model systematically under-predicts the size of the spreads for all maturities but especially for medium term debt compared to CEV. In the latter model the CDS predictions are quite good, especially for the junk rated CDS contracts with intermediate and long-term maturities, for which the errors are less than half than those of constant volatility. The superiority persists also in out-of-sample tests, in which the model-predicted CDS spread is compared to the actual observed spread. The importance of this predictor stems from the fact that the CDS market is much more liquid than the bond market and estimates of credit spreads extracted from it are correspondingly less contaminated by liquidity factors.

In what follows we complete the review of the earlier empirical work involving structural models and default risk, which is extended by this paper. A common result of these earlier papers that are based on constant volatility diffusion is the underestimation of the corporate bond yield spreads,[9] the so-called "credit spread puzzle".[10] One possible solution to the puzzle is an alternative stochastic process of asset (or cash flow) dynamics.[11] Motivated by recent empirical evidence that asset volatility is time-varying[12] and that the presence of stochastic volatility and jumps could improve the fitting of credit spreads even in the context of a Merton-type structural model,[13],[14] Elkamhi et al. (2011) try to explain this puzzle in the context of stochastic volatility asset dynamics. They use the two-dimensional Fortet equation approximation to calculate numerically the first passage default probability and then estimate the equity value by assuming that the maturity of the debt is infinite. Although their work is more general in its asset dynamics assumptions, our CEV approach yields analytical solutions for the first-passage default probability and the equity and debt values. From these, we can derive the endogenous default boundary and the optimal capital structure, which are not available in their

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[5] Zhou (2001) and Collin-Dufresne and Goldstein (2001).

[6] Brennan and Schwartz (1978), and more recently Titman and Tsyplakov (2007) are examples of studies that rely on numerical simulations.

[7] Most structural models are univariate and assume a constant riskless rate of interest. Longstaff and Schwartz (1995), Briys and de Varenne (1997), and Collin-Dufresne and Goldstein (2001) use bivariate diffusion models, in which the term structure of interest rates follows the Vacisek (1977) model and the asset value is a constant volatility diffusion. As the empirical work in Chan et al. (1992) shows, the Vacisek model does not fit actual term structure data. Further, Leland and Toft (1996) note that this bivariate diffusion refinement plays a very small role in the yield spreads of corporate bonds.

[8] See also Cox and Ross (1976), Emmanuel and MacBeth (1982), Cox and Rubinstein (1985), and Schroder (1989).

[9] See Elton et al. (2001), Huang and Huang (2012) and Eom et al. (2004)

[10] See Chen et al. (2009).

[11] See Collin-Dufresne and Goldstein (2001) for the mean reverting leverage ratio, Sarkar and Zapatero (2003) for mean reverting cash flow, Leland (1998) for risk hedging with two risk levels, Huang and Huang (2012) for the double exponential jumps and Elkamhi et al. (2011) for the stochastic volatility.

[12] See Choi and Richardson (2009).

[13] See Zhang et al. (2009).

[14] The Merton-type structural model assumes that the default event only occurs at the maturity of the zero-coupon debt.