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A new approach to assessing model risk in high dimensions ☆

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ABSTRACT

A central problem for regulators and risk managers concerns the risk assessment of an aggregate portfolio defined as the sum of d individual dependent risks X_i. This problem is mainly a numerical issue once the joint distribution of $(X_1, X_2, \dots, X_d)$ is fully specified. Unfortunately, while the marginal distributions of the risks X_i are often known, their interaction (dependence) is usually either unknown or only partially known, implying that any risk assessment of the portfolio is subject to model uncertainty.

Previous academic research has focused on the maximum and minimum possible values of a given risk measure of the portfolio when only the marginal distributions are known. This approach leads to wide bounds, as all information on the dependence is ignored. In this paper, we integrate, in a natural way, available information on the multivariate dependence. We make use of the Rearrangement Algorithm (RA) of Embrechts et al. (2013) to provide bounds for the risk measure at hand. We observe that incorporating the information of a well-fitted multivariate model may, or may not, lead to much tighter bounds, a feature that also depends on the risk measure used. In particular, the risk of underestimating the Value-at-Risk at a very high confidence level (as used in Basel II) is typically significant, even if one knows the multivariate distribution almost completely.

Our results make it possible to determine which risk measures can benefit from adding dependence information (i.e., leading to narrower bounds when used to assess portfolio risk) and, hence, to identify those situations for which it would be meaningful to develop accurate multivariate models.

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1. Introduction

The risk assessment of high dimensional portfolios $\mathbf{X}$ is a core issue in the regulation of financial institutions and in quantitative risk management. In this regard, one usually attempts to measure the risk of the aggregate portfolio (defined as the sum of individual risks X_i) using a risk measure, such as the standard deviation or the Value-at-Risk (VaR). It is clear that solving this problem is mainly a numerical issue once the joint distribution of $\mathbf{X} := (X_1, X_2, \dots, X_d)$ is completely specified. Unfortunately, estimating a multivariate distribution is a difficult task, and thus the assessment of portfolio risk is prone to model misspecification (model risk). At present, there is no generally accepted framework for quantifying model risk. A natural way to do so consists in finding the minimum and maximum possible values of a chosen risk measure evaluated in a family of candidate models. For example, Cont (2006) found bounds on prices of contingent claims, incorporating model risk on the choice of the risk neutral measure used for pricing. In the same spirit, Kerkhof et al. (2010) assessed model risk in the context of management of market risk by computing the worst-case VaR across a range of models chosen based on econometric estimates involving past and present data. A related approach can be found in Alexander and Sarabia (2012), where the authors compare VaR estimates of the model actually used with those of a benchmark model (i.e., the regulatory model) and use the observed deviations to estimate a capital charge supplement to cover for model risk.

More recently, Embrechts et al. (2013) proposed the Rearrangement Algorithm (RA) to find (approximate) bounds on the VaR of high dimensional portfolios, assuming that marginal distributions of the individual risks are known (or prone to negligible model risk) and that the dependence structure (also called the copula) among the risks is not specified. This assumption is

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natural, as fitting the marginal distribution of a single risk X_i $(i = 1, 2, \ldots, d)$ can often be performed in a relatively accurate manner, whereas fitting a multivariate model for $\mathbf{X}$ is challenging, even when the number of observations is large. The bounds derived by Embrechts et al. (2013) are wide, as they neglect all information on the interaction among the individual risks. In this paper, we propose to integrate in a natural way the information from a fitted multivariate model.

Standard approaches to portfolio modeling use a multivariate Gaussian distribution or a multivariate Student's t distribution; however, there is ample evidence that these models are not always adequate. Specifically, while the multivariate Gaussian distribution can be suitable as a fit to a dataset "as a whole," this approach is usually a poor choice if one wants to use it to obtain accurate estimates of the probability of simultaneous extreme ("tail") events, or if one wants to estimate the VaR of the aggregate portfolio $S = \sum_{i=1}^{d} X_i$ at a given high confidence interval; see e.g., McNeil et al. (2010). There is recent literature dealing with the development of flexible multivariate models that allow a better fit to the data. However, no model is perfect, and while such developments are needed for an accurate assessment of portfolio risk, they are only useful to regulators and risk managers if they are able to significantly reduce the model risk inherent in risk assessments.

In this paper, we develop a framework that allows for practical quantification of model risk. Our results make it possible to identify risk measures for which the additional information of a well-fitted multivariate model reduces the model risk significantly, making these measures meaningful candidates for use by risk managers and regulators. In particular, we observe from numerical experiments that the portfolio VaR at a very high confidence level (as used in the current Basel regulation) might be prone to such a high level of model risk that, even if one knows the multivariate distribution nearly perfectly, its range of possible values remains wide. In fact, one may then not even be able to reduce the model risk as computed in Embrechts et al. (2013), where no information on the dependence among the risks is used.

The idea pursued in our approach is intuitive and corresponds to real-world situations. Let us assume that we have N observations for $\mathbf{X}$, i.e. our dataset consists of N vectors of dimension d, $\{\mathbf{x}_i\}_{i=1,\ldots,N}$ where $\mathbf{x}_i = (x_{i1}, \ldots, x_{id})$. We also assume that a multivariate model has already been fitted to this dataset. This fitted distribution is a candidate joint distribution of $\mathbf{X}$ (benchmark model). However, we are aware that the model is subject to misspecification, and we split $\mathbb{R}^d$ into two disjoint subsets: $\mathcal{F}$ will be referred to as the "fixed" or "trusted " area and $\mathcal{U}$ as the "unfixed" or "untrusted" area. Specifically, $\mathcal{U}$ reflects the area in which the fitted model is not considered appropriate. Note that

$$\mathbb{R}^d = \mathcal{F} \bigcup \mathcal{U}, \quad \mathcal{F} \bigcap \mathcal{U} = \emptyset.$$

If one has perfect trust in the model, then all realizations of $\mathbf{X}$ reside in the "trusted" part $(\mathcal{U} = \emptyset)$ and there is no model risk. By contrast, $\mathcal{F} = \emptyset$ when there is no trust in the fit of the dependence, which corresponds to the case studied by Embrechts et al. (2013).

A closely related problem has already been studied for two-dimensional portfolios $(d = 2)$ when some information on the dependence (copula) is available; see e.g., Tankov (2011) and Bernard et al. (2012). Tankov (2011) uses extreme dependence scenarios to find model-free bounds for the prices of some bivariate derivatives, whereas Bernard et al. (2014) use such scenarios to determine optimal investment strategies for investors with state-dependent constraints. While both applications show that finding bounds on copulas in the bivariate case can be of interest, portfolio risk management involves more than two risks. Unfortunately, finding bounds on copulas in the general d-dimensional case in the presence of constraints is not only more difficult but also less useful for risk management applications. The reason is that when $d > 2$, in most cases, the worst copula (under constraints) of a vector $\mathbf{X}$ does not give rise to the highest possible value of the risk measure at hand of $S = \sum_{i=1}^{d} X_i$, because the marginal distributions also have an impact; see e.g., Bernard et al. (2014) for illustrations of this feature.

Hence, in this paper we study bounds for risk measures of the aggregate risk S by using information on the *multivariate* joint distribution of its components X_i (which embeds information on the dependence) rather than using copula information. Some previous papers have dealt explicitly with the presence of (partial) information on the dependence structure: Embrechts and Puccetti (2010) and Embrechts et al. (2013) consider the situation in which some of the bivariate distributions are known; Denuit et al. (1999) study VaR bounds assuming that the joint distribution of the risks is bounded by some distribution; and Bernard et al. (2015) compute VaR bounds when the variance of the sum is known. However, the setup in these papers is often difficult to reconcile with the information that is available in practice; or, it does not make use of all available dependence information. Furthermore, while the bounds that are proposed in these papers might be sharp (attainable), they do not always make it possible to strengthen the unconstrained[1] bounds in a significant way and are often difficult to compute numerically, especially for higher dimensions with inhomogeneous risks.

The paper is organized as follows. We lay out our setting in Section 2. Sections 3 and 4 are devoted to the development of a practical method for deriving bounds on risk measures. This method relies on a (discretized) matrix representation of the portfolio $\mathbf{X}$ and builds on the Rearrangement Algorithm that was recently developed by Puccetti and Rüschendorf (2012) and further studied by Embrechts et al. (2013). We illustrate the results using various examples. In Section 5 we provide two applications. First, we find the minimum variance portfolio under model uncertainty, and next we assess the model risk of a credit portfolio.

The numerical results show that the proposed bounds, which take into account dependence information, typically outperform the (unconstrained) ones already available in the literature and thus allow for more realistic assessment of model risk. However, model risk remains a significant concern, especially when using a risk measure that focuses on "tail type" events, such as the VaR computed at very high confidence level.

2. Setting

Let $\mathbf{X} := (X_1, X_2, \ldots, X_d)$ be some random vector of interest having finite mean and defined on an atomless probability space. Let $\mathcal{F} \subset \mathbb{R}^d$ and $\mathcal{U} = \mathbb{R}^d \setminus \mathcal{F}$. We assume that we know.

(i) the marginal distribution F_i of X_i on $\mathbb{R}$ for $i = 1, 2, \ldots, d$,
(ii) the distribution of $\mathbf{X} \mid \{\mathbf{X} \in \mathcal{F}\}$,
(iii) the probability $p_{\mathcal{F}} := P(\mathbf{X} \in \mathcal{F})$ and $p_{\mathcal{U}} := P(\mathbf{X} \in \mathcal{U}) = 1 - p_{\mathcal{F}}$.

Without loss of generality, we can assume that $\mathcal{F} = \emptyset$ if and only if $p_{\mathcal{F}} = 0$. As the joint distribution of $\mathbf{X}$ is only fully specified on the subarea $\mathcal{F}$ of $\mathbb{R}^d$, risk measures (e.g., the VaR) of the aggregate risk $\sum_{i=1}^{d} X_i$ cannot be computed precisely (unless $p_{\mathcal{F}} = 1$). In fact, there are many vectors $\mathbf{X}$ that satisfy the properties (i), (ii) and (iii) but have a different risk measure of their sum. In order to derive the maximum (minimum) possible value, it is convenient to consider a mixture representation. Specifically, consider the indicator variable $\mathbb{I}$ corresponding to the event "$\mathbf{X} \in \mathcal{F}$"

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[1] Here, "unconstrained bounds" refers to bounds that are obtained when only the marginal distributions are fixed, but no dependence information is used.

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