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Pricing and static hedging of American-style knock-in options on defaultable stocks ☆

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ABSTRACT

This paper applies the *static hedge portfolio* approach (SHP) of Chung et al. (2013) in two new directions. First, the SHP approach is generalized from the *constant elasticity of variance* (CEV) model of Cox (1975) to the *jump to default extended CEV* (JDCEV) framework of Carr and Linetsky (2006). For this purpose, the recovery value of the American-style down-and-in put is hedged through the one attached to a European-style plain-vanilla contract whereas for an up-and-in put it is necessary to use the recovery component of the corresponding European-style up-and-in option. Second, the SHP methodology is adapted from single to double barrier American-style knock-in options by matching the value of the hedging portfolio along both lower and upper barriers. Finally, and to benchmark the accuracy of the novel SHP pricing solutions, the *optimal stopping approach* of Nunes (2009) is also extended to price American-style double knock-in options under the JDCEV framework. Such extension highlights the relevant credit derivative component embedded in American-style knock-in equity puts.

1. Introduction

Barrier option contracts are non-standard (or exotic) options that are widely used by institutional investors, banks, and corporations in their risk management activities. A typical barrier option contract has a single trigger clause (in the case of single barrier options) or two threshold provisions (for double barrier options). Contrary to a plain-vanilla option, the payoff of a barrier option is contingent upon whether the underlying asset price has reached the specified barrier(s) at some earlier point during the option's lifetime. *Knock-in* options are triggered into existence only when the underlying asset price ever crosses the barrier level(s), whereas *knock-out* options cease to exist if a barrier is touched before the option's maturity.

The inclusion of American-style features gives the option's holder the additional flexibility of early exercise but, to the authors' knowledge, there are no organized markets yet for American-style barrier options. Nevertheless, American-style barrier options can still be traded in the over-the-counter market since the previously described knock-in/out event methodology as well as the procedures related to the early exercise of equity barrier options are all regulated by the 2011 ISDA (International Swaps and Derivatives Association, Inc.) Equity Derivatives Definitions and Appendix. Moreover, these exotic contracts can also be used as the building block of many equity-linked structured products, which are extensively issued by the banking industry.[1] And since the issuing banks generally also act as market makers for their own products in the secondary market, accurate and efficient hedging (and pricing) tools must be found under a sufficiently general and realistic model setup. For the case of American-style knock-in equity options, this paper will show that such level of generality must encompass the bankruptcy risk associated to the underlying equity.

The valuation of European-style barrier options is already well established in the literature under alternative underlying asset price dynamics—see, for instance, Merton (1973), Rubinstein and Reiner (1991), Rich (1994), Kuan and Webber (2003), or Sbuelz (2005) under the *geometric Brownian motion* (hereafter, GBM) assumption, and Boyle and Tian (1999), Davydov and Linetsky (2001), Davydov and Linetsky (2003) and Mijatović and Pistorius

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[1] For instance, Stoimenov and Wilkens (2005) examine equity-linked structured products on the German stock index DAX and on the 30 individual stocks from this index, and find that the majority of the exotic option components embedded in the structured products on individual stocks are of the knock-in type.

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(2013) under the *constant elasticity of variance* (hereafter, CEV) model of Cox (1975). However, and similarly to the standard American-style options case, one has to resort to numerical methods or analytical approximations for pricing (and hedging) barrier option contracts possessing early exercise features. Such numerical techniques include lattice schemes—such as the trinomial methods of Ritchken (1995) and Cheuk and Vorst (1996), the finite difference schemes of Boyle and Tian (1998) and Zvan et al. (2000), or the generalized binomial model of Chung and Shih (2007)—as well as the Monte Carlo approach of Glasserman and Staum (2001) and the quadrature methods of, for instance, Andricopoulos et al. (2003) or Chung et al. (2010).

Although all these valuation methods are typically very flexible and can potentially be used to price single and double barrier options with time-varying trigger clauses, they are still numerical approaches often requiring an intensive computational burden. This issue is even more critical for the valuation of knock-in options with lattice schemes since a complete tree must be constructed for such contracts.[2] And, even though some analytical approximations have already been proposed for knock-out options—see, for instance, the *integral representation approach* adopted by Gao et al. (2000)—the literature on the valuation of American-style knock-in options is much more scarce. Additionally, it is well known that while the sum of the prices of European-style knock-in and knock-out barrier options is equal to the price of a standard European-style option, such *in-out barrier parity relation* cannot be applied to American-style options—as argued by Carr et al. (1998, p. 1169), Dai and Kwok (2004, p. 187), or Chung et al. (2013, p. 191).

To the best of our knowledge, only a few attempts have been made for valuing American-style knock-in options through analytical solutions. Haug (2001) uses the well-known reflection principle, but restricts the focus to the special case in which the striking price is always above the knock-in barrier level. Dai and Kwok (2004) generalize the analysis by considering the various possibilities of interaction between the knock-in and the exercise regions of the option contract. AitSahlia et al. (2004) propose both a modified binomial algorithm and an analytical approximation, being the latter based on the price decomposition into a European-style option and an early exercise component (evaluated through a Gaussian quadrature).

While all these aforementioned contributions were important for giving new insights into the valuation of American-style barrier options, they generally possess two common undesirable features: (i) they are limited to the case where the underlying asset price follows a GBM process; and/or (ii) they only cope with American-style single barrier knock-in options. Chung et al. (2013) constitutes a notable exception for tackling the first modeling limitation by extending the *static hedge portfolio* (hereafter, SHP) approach of Derman et al. (1995) and Carr et al. (1998) to price and hedge American-style (but single barrier) knock-in options, under both the GBM and (restricted) CEV models.[3]

This paper contributes to the option pricing literature in three ways. First, the SHP methodology is applied to double barrier American-style knock-in equity options, accommodating both endogenous bankruptcy and time-dependent barriers, by matching the value of the hedging portfolio along both lower and upper barriers. Second, the SHP approach of Chung et al. (2013) is generalized from the CEV model of Cox (1975) to the *jump to default extended CEV* (hereafter, JDCEV) framework of Carr and Linetsky (2006), which is able to accommodate, as special cases, the CEV and standard GBM diffusion processes. This is not a trivial or straightforward extension because the boundary conditions that now must be satisfied by the American-style barrier knock-in (put) option in the presence of a possible jump to default event are more complex than the ones attached to the simpler context of a pure diffusion process. Moreover, we consider two different recovery assumptions for the American-style barrier put contracts, namely: *Recovery at the default time* and *recovery at the maturity date*, as used by Nunes (2009) and Ruas et al. (2013), respectively, in the context of plain-vanilla American-style options.

Finally, the *optimal stopping approach* of Nunes (2009) is also extended to value American-style double barrier knock-in options under the general JDCEV framework. This contribution is used not only to test the accuracy of the novel SHP pricing solutions proposed but also to highlight the credit derivative component embedded in American-style knock-in equity puts. More formally, any American-style down-and-in put with a sufficiently low barrier level will be shown to be, essentially, a unit recovery claim—i.e. a contract that pays one monetary unit when and only when default occurs before the contract's expiry date. In other words, and for low barrier levels, American-style down-and-in puts behave more as a credit derivative rather than as an equity derivative, which fully justifies the choice of an hybrid credit-equity valuation model, such as the JDCEV process, for the pricing of these exotic contracts. The intuition is that the American-style down-and-in put is always early exercised whenever the stock price drops to the (low) barrier level—as long as such barrier level is strictly below the early exercise boundary of the corresponding plain-vanilla put—and such sharp decline of the stock price should be associated to the default of the underlying equity. Note that default may take place at strictly positive stock price levels, and the rational for this stylized fact can be found in the strategic default literature—see, for instance, Carr and Wu (2011, p. 475) and the references therein.

The novel pricing solutions proposed will be shown to be extremely accurate and easy to implement, and should prove useful to researchers and practitioners in credit and equity derivatives markets, because, and as noted by Ruas et al. (2013, p. 4060), the JDCEV model is consistent with three stylized facts, namely: The existence of an inverse relation between stock returns and realized volatility (*leverage effect*), as observed, for instance, by Black (1976) and Bekaert and Wu (2000); the negative correlation between the implied volatility and the strike price of an option contract (*implied volatility skew*), as documented, for example, in Dennis and Mayhew (2002); and the positive correlation between default indicators and equity volatility, documented, for instance, in Campbell and Taksler (2003).

The remainder of this article is organized as follows. Section 2 summarizes our modeling assumptions as well as the already known plain-vanilla option pricing solutions under the JDCEV framework. Our main results are contained in Sections 3 and 4, where up-and-in, down-and-in, and double knock-in American-style options are priced under the JDCEV model and through both the SHP and optimal stopping approaches. These novel pricing formulae are implemented in Section 5, and Section 6 presents the main conclusions. All accessory proofs are relegated to the Appendix.

2. JDCEV framework

The financial model adopted for the valuation of American-style knock-in equity options is the unified framework proposed by Carr and Linetsky (2006) for the consistent pricing of corporate

[2] For example, to price a down-and-in (up-and-in) option one actually has to calculate the value of a plain-vanilla American option at every node just below (above) the barrier. This is not necessary for knock-out options due to the existence of knock-out thresholds.

[3] Note that Chung et al. (2013) restrict the CEV model for an elasticity parameter (β) equal to $4/3$, which allows the complementary noncentral chi-square distribution function to be computed through the standard normal probability law. However, and as shown by Larguinho et al. (2013), both the option prices and "Greeks" formulae can be easily and efficiently computed under an unrestricted CEV model accommodating both direct and indirect leverage effects typically observed across a wide variety of options markets.