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Rehypothecation dilemma: Impact of collateral rehypothecation on derivative prices under bilateral counterparty credit risk

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ABSTRACT

Rehypothecation is the practice where a derivatives dealer reuses collateral posted from its end user in over-the-counter (OTC) derivatives markets. Although rehypothecation benefits the end user through cost reduction of derivative trades, it also creates additional counterparty credit risk since the end user may not receive the collateral back when the dealer suddenly defaults. To evaluate the benefits and risks of rehypothecation, we propose a derivative pricing framework with bilateral counterparty credit risk that determines the amount of rehypothecable collateral. We also model the realistic features of derivative trades: two different types of collateral, the time delay of collateral posting and the rating-dependent collateral agreement. We apply our pricing framework to cross currency swaps and investigate the impact of rehypothecation on the swap spreads.

1. Introduction

Collateral plays two key roles in over-the-counter (OTC) derivative trades. The first role is credit mitigation. A derivative dealer reduces counterparty credit risk with collateral posted from a derivative end user. This role is well understood and widely documented. For example, Bliss and Kaufman (2006) document that the use of collateral as a credit mitigation tool has enabled OTC derivatives markets to expand. The second, less understood role is rehypothecation, the reinvestment of collateral posted from an end user. A derivative dealer makes a profit by reusing his client's collateral. That part of the profit is then reflected in the pricing of derivatives in order to reduce the cost of derivative trades for the end user. The total amount of collateral received that can be rehypothecated by the largest US banks is huge. According to Singh (2010), it was about 2 trillion dollars as of 2009.

Rehypothecation is off-balance-sheet leverage that creates additional counterparty credit risk since the end user may not receive collateral back when the dealer suddenly defaults. It is currently criticized as a major part of shadow banking system. In fact, Fender and Gyntelberg (2008) document that a number of hedge funds that posted collateral to Lehman Brothers were not able to receive the collateral when Lehman Brothers went bankrupt. This occurred because Lehman Brothers reused it as its own collateral. Some end users are now requesting their dealer to segregate their collateral from the dealer's own assets, although the segregation of collateral would increase the cost of derivative trades for them.[1] Sawyer (2010) calls that trade-off the "rehypothecation dilemma".

To the best of our knowledge, there have been no studies evaluating the benefit and risk of rehypothecation in derivative pricing. We aim to fill this gap by proposing a derivative pricing framework with bilateral counterparty credit risk and collateral rehypothecation. In our pricing framework, two parties post collateral to each other and then a derivative dealer reinvests some proportion of its end user's collateral. We assume that a derivative dealer can earn a positive excess return from rehypothecation by exposing himself to additional default risk. In addition, we make an important assumption that the benefit from rehypothecation is fully reflected in the price of a derivative trade. Given those assumptions, the cost of derivative trades decrease and default risk of a derivative dealer increases, as a derivative dealer rehypothecates a larger amount of collateral.

We incorporate some realistic features of counterparty credit risk management to the pricing framework described above. First and foremost, we distinguish between Variation Margin and Initial

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[1] Regulatory reform of rehypothecation in OTC derivative markets including the ban on rehypothecation is currently under discussion. See Acharya et al. (2010) for more details.

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Margin. Variation Margin is re-estimated and posted on a daily basis, while Initial Margin is posted at the start of a trade and is kept constant over time. It is important to take into account these two different types of collateral, because Initial Margin is held by a dealer, even if the mark-to-market (MTM) value is positive from the view of end user. In fact, ISDA (2010) reports that significant loss arose from over-collateralized Lehman Brothers through the provision of Initial Margin.[2] Furthermore, regulatory requirements for these two types of collateral are different in newly proposed regulations.

We also model other realistic features such as the time delay of collateral posting and the rating-dependent collateral agreement. Those two features are the key ingredient for counterparty credit risk management. As for the former, a derivative dealer sets Initial Margin in order to cover a residual risk that arises due to time delay of collateral posting. As for the latter, it is common to make the amount of collateral posted from a counterparty depend on its credit rating. For example, Acharya (2011) reports that large derivative dealers need to post a huge amount of collateral to their counterparties, if they are downgraded significantly.

Our pricing framework can be seen as a generalized version of Johannes and Sundaresan (2007) model[3] They show that one can adjust a discount rate by subtracting the excess return of rehypothecation collateral from a risk-free rate if the MTM value of a derivative trade is always perfectly collateralized. In our pricing framework, such an adjusted discount rate does not work since the MTM value is not perfectly collateralized due to time lag of collateral posting. Plus, a dealer is over-collateralized by Initial Margin at the start of a trade.

We employ a jump-to-default model to accommodate those features in a tractable way. Jump-to-default models are recently studied in Linetsky (2006), Carr and Linetsky (2006), Campi et al. (2009), and Mendoza-Arriga and Linetsky (2011). There are two differences between those previous studies and our model. First, we incorporate credit rating migration following Albanese and Chen (2006). Second, we associate a jump-to-default risk of a bank with the rehypothecation rate that determines the proportion of collateral rehypothecated by a bank. In doing so, we can capture the risk of rehypothecation caused by off-balance-sheet leverage.

As a theoretical contribution, we also develop the method to compute the probability that a counterparty has a high credit rating when the last margin call is made before it defaults. This mathematical problem arises when we calculate potential future exposure under rating-dependent collateral agreement. In previous literature, Collin-Dufresne and Goldstein (2001) apply the method proposed by Fortet (1943) to calculate the distribution of first hitting time in the context of a structural model of default risk. We combine Fortet's (1943) method with the result on the last exit time obtained in Salminen (1988). The density function of the last exit time is needed to compute the probability of the last time of downgrade before default time.

We apply our pricing framework to cross currency swaps and investigate the impact of rehypothecation on the coupon payments that an end user receives. Specifically, we numerically analyze the relationship between the rehypothecation rate and the additional spreads produced through reinvestment of collateral. We find the additional spreads increase with increasing rehypothecation rate under plausible model parameter setting. We also show that there exists approximately a linear relationship between them. However, the relationship becomes increasingly nonlinear as a jump-to-default risk is more sensitive to the rehypothecation rate.

We also report that the relationship between the sensitivity of a jump-to-default risk to a rehypothecation rate substantially differs depending on the maturity of a cross currency swap. For long-term swaps, a higher sensitivity lowers the benefit from rehypothecation. This result is intuitive since the duration of collateral reinvestment is shortened by an increased jump-to-default risk. We refer to this effect as duration effect. By contrast, for short-term swaps, the opposite is the case; A higher sensitivity raises the benefit. The reason for this counterintuitive result is that a higher sensitivity increases the default risk. As a results, a dealer is more likely to obtain Initial Margin, even if the MTM value of a trade is close zero. The benefit from Initial Margin is reflected in the price of a derivative trade. Hence, the cost of a derivative trade is smaller when the sensitivity is high. This over-collateralization effect dominates duration effect in short-term swaps.

The rest of paper is organized as follows. Section 2 describes our derivative pricing framework with rehypothecation. Section 3 models default events and credit rating migration and incorporate the risk of rehypothecation. Section 4 applies the model to pricing cross currency swaps and analyze the impact of rehypothecation on the swap spreads. Section 5 concludes.

2. A pricing framework for collateralized derivatives

We denote a bank and its counterparty with b and c, respectively. In our setting, the bank b is a dealer of an underlying derivative trade, while the counterparty c is an end user who enters into a derivative trade with the bank b, such as a hedge fund and a corporation. We call the counterparty c an end user to avoid ambiguity. We valuate mark-to-market (MTM) derivative prices V_t from the view of the bank b. Both two parties have credit rating s_t^i $(i = b, c)$ at the time t.

We model two different types of collateral; Variation Margin M_t and Initial Margin A. Variation Margin M_t varies on a daily basis and covers daily fluctuation of mark-to-market (MTM) value of a derivative trade V_t, while Initial Margin A is kept constant and reduces the residual risk which is not covered by Variation Margin M_t due to the time delay of collateral posting and an unexpected default of the counterparty.

In the first subsection, we model Variation Margin and then discuss our specification of Initial Margin. In the second subsection, we provide a pricing framework with collateral rehypothecation. Fig. 1 graphically describes the structure of the pricing framework which we explain below.

We assume that there exists an equivalent martingale measure $\mathbb{Q}$ and a derivative trade V_t is priced under this measure. We will define default events in Section 3.

2.1. Uncollateralized exposure and Initial Margin

We describe the lagged collateral model based on Pykhtin (2010), because it is realistic to assume that there is the time lag between the last margin call made before default and the settling of the trades with the defaulting counterparty.[4] We denote the time lag with δt. Under the current market practice, the time lag δt is set equal to ten business days or two weeks. Hence, we set $\delta t = 1/24 (\approx 0.042)$.

The one-way collateral agreement on Variation Margin M_t is given by

$$M_t = \max(V_{t-\delta t} - K^c, 0). \tag{2.1}$$

In practice, two-way collateral agreement is more widely used than one-way collateral agreement. The two-way collateral agreement is

[2] In ISDA (2010), Initial Margin is called Independent Amount.

[3] Other relevant studies include, but not limited to, Fujii and Takahashi (2010), Piterbarg (2010), Brigo et al. (2011), and Burgard and Kjaer (2011). Yet, Initial Margin is not explicitly modeled in these studies.

[4] The time delay δt is called Margin Period of Risk among practitioners of counterparty credit risk management. See the Chapter 9 in Gregory (2010).

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