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Semiparametric estimation of multi-asset portfolio tail risk

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ABSTRACT

When correlations between assets turn positive, multi-asset portfolios can become riskier than single assets. This article presents the estimation of tail risk at very high quantiles using a semiparametric estimator which is particularly suitable for portfolios with a large number of assets. The estimator captures simultaneously the information contained in each individual asset return that composes the portfolio, and the interrelation between assets. Noticeably, the accuracy of the estimates does not deteriorate when the number of assets in the portfolio increases. The implementation is as easy for a large number of assets as it is for a small number. We estimate the probability distribution of large losses for the American stock market considering portfolios with ten, fifty and one hundred assets of stocks with different market capitalization. In either case, the approximation for the portfolio tail risk is very accurate. We compare our results with well known benchmark models.

1. Introduction

Multi-asset managers hedge risk by investing in assets that ideally have low correlations. Unfortunately through markets integration and financial crises, correlations between assets have become more often positive and stronger. Expressions like “no place to hide” are now commonplace in the finance and investments news. In such environment it is paramount for the multi-asset manager to accurately monitor the tail risk of his portfolio.

The risk of incurring very large losses in a financial position is commonly measured by Value-at-Risk (VaR) or Expected Shortfall (ES). The computation of tail risk measures boils down to the estimation of the probability of observing very large losses. The main challenge presented by this task comes from the fact that large losses are by their own nature scarce. With few observations it is not trivial to implement successfully statistical methods.

Classical finance theory assumes that asset returns have a multivariate normal distribution. Yet, the normality assumption leads often to the underestimation of the probability of large losses. As a natural extension, the Student-*t* distribution allows for heavy-tails which help to overcome the “optimism” of the normal model relating the frequency of large losses. The drawback of the Student-*t* model is that financial returns are typically asymmetric due to many small gains and few large losses. The Student-*t* model is symmetric, hence assuming that the distribution of losses and gains have the same shape is often unreasonable. Another well known model is a mixture of normal distributions. We include a mixture of normal distributions as a benchmark model in our study.

In fact, to estimate tail risk we only need a good model for the (long) tail of the distribution of the losses. Extreme Value Theory (EVT) models focus on the tail of the distribution only. Hence, EVT provides a tailor made methodology to estimate the probability of large losses.

Multi-asset portfolios exist because managers want to take advantage of the interaction between assets. This is important from the modeling point of view. Portfolio losses are the result not only of the individual assets performance but also, and very importantly, from the interaction between assets. This raises the question of the choice between univariate and multivariate models for the tail risk. Multivariate models make use of information from individual assets while univariate models use the information already synthesized in the portfolio returns. Hence, from the accuracy point of view ideally we would prefer the multivariate approach.

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Multivariate EVT models exist. These are parametric models where a dependence structure for the large losses of several assets is imposed. However, since large losses are rare, parameter estimation and specification tests become a serious challenge. Our solution is to use a semiparametric EVT estimator for computing portfolio tail risk. This estimator captures the statistical tail features of each individual asset as well as their interaction, without imposing a specific dependence structure. In addition it can be used with high-dimensional portfolios.

The probability of multi-asset portfolio losses is used in several contexts by managers and analysts as in portfolio selection with downside risk, and in particular for estimating VaR. The advent of modern finance crystalized the need of modeling large losses in multi-asset positions. The safety-first principle of Roy (1952) constrains portfolio selection on a small probability of large losses. Portfolio selection with downside risk constraints and risk management practice have often used VaR as the risk measure for large losses. Traditionally, these probabilities and risk measures have been computed based on the assumption that asset returns have a multivariate normal distribution. Indeed, if asset returns are assumed to be multivariate normal then closed form solutions can be used for multi-asset portfolio VaR and the calculation of constrained portfolio weights; see for instance Alexander and Baptista (2002) and Alexander and Baptista (2008). However, risk management with VaR constraints (VaR-RM) leads one to select positions with higher variance. Furthermore, as Basak and Shapiro (2001) show, when losses larger than VaR occur they are larger if VaR-RM is used than if VaR-RM is not used. All these findings argue for good models and estimators for the tail of the loss distribution.

Assuming a multivariate normal distribution for asset price returns implies that, asymptotically, large losses of individual assets occur independently. In contrast, especially in periods of crisis, we observe that several financial assets can suffer simultaneously dramatic losses. The fact that financial data often does not support the hypothesis of a normal distribution for asset returns suggests the use of a more appropriate model. The problem with using an alternative approach is that closed forms for VaR or ES are not known for most probability distributions. One possible alternative is to use Tchebychev's inequality to approximate the tail of a portfolio distribution. But Arzac and Bawa (1977), for instance, found that this inequality produces poor approximations. As a response, Arzac and Bawa (1977) use a nonparametric model for the asset returns distribution. Later, Gourieroux et al. (2000) also model the portfolio returns distribution using a nonparametric model. However, there is no need to model the entire distribution in order to obtain the probability of large losses. It is sufficient to model the tail of the distribution. EVT was developed specifically for modeling large events. Hence, EVT provides a set of tools appropriate to model the probability of portfolio large losses.

EVT has been used in finance mainly with univariate models. Univariate EVT models have been used in different settings by numerous authors. For instance, Jansen et al. (2000), Longin (2000) and McNeil and Frey (2000) compute portfolio VaR by modeling the portfolio returns with univariate EVT. Susmel (2001) implements portfolio selection with safety first using univariate EVT to model the portfolio large losses. Zhang and Shinki (2007) use univariate EVT to model high frequency asset returns. Brooks et al. (2005) use univariate EVT models and compares them with other approaches to model large return losses.

Although the use of univariate EVT models can result in an important loss of information, multivariate EVT models are far less used in finance. Hyung and de Vries (2007) reduce the multivariate problem to the univariate case by modeling the asset returns with a one factor model and modeling the (independent) idiosyncratic component of the tail risk with an EVT model. Longin (2000) computes the VaR of multivariate positions using a decomposition into a set of risk factors. He models the extreme dependence of risk factors as a linear combination of the dependence function of total dependence and asymptotic dependence. Daul et al. (2003) use the grouped *t*-copula to capture the risk in a large set of risk factors in an application to credit risk. Other authors, as Ang and Chen (2002), capture the multivariate structure by modeling correlations with regime switching models. Specifically, Tsafack and Garcia (2011) use an EVT copula in a regime-switching copula model to capture the joint extreme behavior of asset returns. Multivariate EVT models are also implemented by Longin and Solnik (2001) and by Poon et al. (2004). In these articles the logistic function is used to model the dependence of extreme asset returns. These models are completely parametric and the estimation of their parameters is usually difficult due to the lack of historical (extreme) observations.

In this article we estimate the tail risk of multi-asset portfolios using a semiparametric EVT approach. We use an estimator which assumes a parametric EVT specification for the univariate asset tail risk combined with a nonparametric estimator for the dependence structure of the portfolio large losses. This reduces the problem of model misspecification, increasing the accuracy of the results compared with fully parametric models. Another advantage of the methodology is that the estimation of the portfolio tail risk does not become more difficult as the number of assets in the portfolio increases.

To the best of our knowledge this method has not been used before in tail risk estimation. The contribution of this article has two aspects. The estimation of the tail of the distribution of the portfolio large losses, and the specific estimation of the portfolio tail risk using VaR as a measure of risk.

We estimate the tail of three equity portfolios. The first portfolio has ten assets, the second portfolio has fifty assets and the third portfolio has one hundred assets. We compare the performance of the semiparametric estimator with the performances of a mixture of normal distributions, and of a Student-*t* model.

For the three portfolios described above we estimate tail risk with VaR computed at several levels of confidence. The accuracy of the tail risk estimates is inspected using a backtesting procedure.

This paper is organized as follows. Section 2 summarises the theoretical foundation of the semiparametric estimator for tail risk. Section 3 describes the statistical estimation methodology. In Section 4 we estimate the distribution of large losses of the three equity portfolios, and the portfolios tail risk. Section 5 concludes this article.

2. The semiparametric estimator for portfolio large losses

Very large movements of asset prices, although often implying financial distress, do not occur frequently. Modeling rare losses is a challenging task from the statistical point of view. The natural tool to quantifying risk in positions involving assets subject to rare events is EVT. Here we focus on quantifying risk of portfolios with possibly a large number of assets.

EVT provides a model for the multivariate distribution of the maximum return[1] observed for each asset over a period of time. We denote by $\mathbf{R} = (R_1, R_2, \ldots, R_d)$ the random vector representing the one period loss returns of d assets. A main result in extreme value theory is that for the distributions commonly used for financial returns, there is a limit distribution for the componentwise maxima. In the following we summarize this result and show how to use consequent results in quantifying multi-asset portfolio large losses.

[1] Often we are interested in modeling the minimum return but this is equivalent to modeling the maximum loss. Hence the results presented for the maximum can be used for the minimum return.