Journal of Banking & Finance 44 (2014) 130–140

Contents lists available at ScienceDirect

Journal of Banking & Finance

journal homepage: www.elsevier.com/locate/jbf

Analytical pricing of discrete arithmetic Asian options with mean reversion and jumps

Shing Fung Chung, Hoi Ying Wong *

Department of Statistics, The Chinese University of Hong Kong, Shatin, Hong Kong

ARTICLE INFO

Article history:
Received 24 July 2013
Accepted 9 April 2014
Available online 19 April 2014

JEL classification:
G12
G13

Keywords:
Asian options
Fourier transform
Mean reversion
Jump diffusion

ABSTRACT

Empirical evidence indicates that commodity prices are mean reverting and exhibit jumps. As some commodity option payoffs involve the arithmetic average of historical commodity prices, we derive an analytical solution to arithmetic Asian options under a mean reverting jump diffusion process. The analytical solution is implemented with the fast Fourier transform based on the joint characteristic function of the terminal asset price and the realized average value. We also examine the accuracy and computational efficiency of the proposed method through numerical studies.

1. Introduction

Asian options are derivatives whose payoff depends on the average of the prices of the underlying asset within a specified time interval. This average can be measured with different monitoring frequencies, such as daily, weekly or monthly. Asian options first appeared in 1987 when the Banker's Trust Tokyo office developed a commercially used pricing formula for options on the average crude oil price, hence the name "Asian" options. Asian options could reduce the risk of market manipulation in the underlying at maturity as the payoff depends on the average of price across time. Furthermore, Asian options are now widely used in the commodity market as a hedging device. Eydeland and Wolyniec (2003) report that many delivery companies in the gas market rely on Asian options for risk management. These kinds of securities are often used in the oil markets to stabilize the cash flows that stem from meeting obligations to clients. Boyle and Boyle (2001) briefly introduce the history and evolution of Asian options.

In addition to being popular in the commodity market, Asian options have attracted a great deal of academic attention. Boyle and Potapchik (2008) provide a summary of the different methods of pricing Asian options and the approaches for computing price sensitivities. The methods discussed include Monte Carlo simulation, finite difference approach and various quasi analytical approaches and approximations. Asian options can be divided into two main categories: the geometric Asian option and arithmetic Asian option. In practice, most Asian options use the arithmetic average. In this case, no closed-form solution exists. Because the distribution of the sum of log normally distributed random variables is analytically intractable, the problem of pricing arithmetic Asian options is computationally challenging even in a rather simple model such as the Black–Scholes model.

When analyzing the price dynamics of commodity products, important empirical features of commodity prices need to be considered. The most prominent feature is the property of mean reversion. Numerous empirical studies support that commodity prices return to their mean level rather than increasing exponentially. Bessembinder et al. (1995) find significant evidence supporting mean reversion in nine commodity markets. In addition, strong patterns have been shown in agricultural commodities and crude oil while weak patterns have been shown in metals. Schwartz (1997) reveals strong mean reversion for both copper and oil. Casassus and Collin-Dufresne (2005) confirm the existence of mean reversion in the crude oil, copper, gold and silver markets. Geman and Roncoroni (2006) discover that the power prices in most US power markets exhibit both the mean reversion effect

* Corresponding author. Tel.: +852 3943 8520; fax: +852 2603 5188.
E-mail addresses: edmondchungsf@gmail.com (S.F. Chung), hywong@cuhk.edu.hk (H.Y. Wong).

http://dx.doi.org/10.1016/j.jbankfin.2014.04.011
0378-4266/

and spikes in trajectories. Numerous studies have examined the effect of mean reversion on option pricing. Wong and Lo (2009) propose the mean reversion stochastic volatility model for option pricing. They derive the analytical solution for European options and introduce the bivariate trinomial lattice approach for path-dependent options. Wong and Zhao (2010) extend the mean reversion model with a stochastic volatility of Wong and Lo (2009) to a multi-scale stochastic volatility model. Ewald et al. (2013) also consider stochastic volatility model and find that Australian options are equivalent to fixed or floating strike Asian options. By studying the two types of option in parallel, much of the problems could be solved effectively. Instead of considering stochastic volatility, we incorporate jumps into the mean reversion for pricing discrete Asian options. Specifically, our model is a jump-diffusion extension of the one proposed by Fusai et al. (2008).

The extended model is motivated by the numerous empirical studies that commodity prices exhibit jumps. Deng (2000) points out that the price dynamics of energy commodities contain jumps, especially when the commodity is costly to store and the demand has low elasticity, for example, electricity. In this case, the marginal cost curve of the electricity supply almost always has a kink at a certain capacity level. Once the demand exceeds that particular level, a jump in price will be formed. Hilliard and Reis (1999) find that adding a jump to soybean prices can thicken the distribution tails of the futures and reduce the price discrepancy for deeply out-of-the-money options. Schmitz et al. (2013) investigate the corn, soybean and wheat markets in the US and show that the addition of a jump component to the spot price dynamic has a significant effect through a Bayesian approach. Many studies have been done on option pricing under jump diffusion process. For example, Fuh et al. (2013) price path-dependent options under a double exponential jump-diffusion model by applying continuity correction. This method is especially well when the jump volatility is small compared to the total volatility.

The remainder of this paper is arranged as follows. In Section 2, we introduce the proposed mean reverting jump diffusion model and the procedures for deriving the joint characteristic function of the terminal asset value and the arithmetic average of the asset values. We then extend this model to better incorporate information available in the market. The extended model is presented in Section 3. The joint characteristic function for the terminal asset value and the arithmetic average of the asset values under the extended model is also derived in this section. Section 4 describes the analytical solution for Asian option prices in terms of the Fourier transform. We also explain the use of the fast Fourier transform technique in calculating the analytical price for Asian options. In Section 5, we examine the performance of the proposed analytical solution through numerical experiments. We first evaluate the accuracy and efficiency of the pricing mechanism by benchmarking against simulation. We then proceed to investigate the price sensitivity in relation to different parameter values and the payoff structure. The averaging effect of Asian options is also studied. Section 6 provides a brief summary of the paper. In Appendix A, we discuss the case of a normal jump size. Finally, we will explore using level dependent jump to generate both upward and downward jumps while maintaining the asset price to be positive.

2. Model with constant parameters

2.1. Model specification

The model of Fusai et al. (2008) considers spot price dynamics as a square root process driven by Brownian motion. We extend this approach to include a jump part in which the jump size follows the exponential distribution. The arrival of jumps is modeled by a Poisson process. We assume the Brownian motion, jump rate and jump size to be independent of each other. This is the CIR model with exponential jump extension. The dynamic under the risk-neutral probability measure is:

$$dS_t = \beta\left(\eta - \frac{\mu\lambda}{\beta} - S_{t-}\right)dt + \sigma\sqrt{S_{t-}}dW_t + JdN_t, \tag{1}$$

where $J \sim Exp(\mu)$ and $N_t \sim Poi(\lambda t)$.

Hoepfner (2009) considers a time inhomogeneous Cox–Ingersoll–Ross diffusion with positive jumps and prove the existence of unique strong solution by imposing restrictions on the jump measure. Beliaeva and Nawalkha (2011) also consider jumps in the CIR model for modeling interest rates and use two kinds of jump distribution: the exponential jump and the lognormal jump. They argue that the exponential jump extension is useful during periods when positive jumps are expected to dominate negative jumps. Alternatively, the lognormal jump is the only known extension in the literature that allows both positive and negative jumps for the CIR model. Although this set up is more realistic, no analytical solution can be deduced. To examine the effects of positive and negative jumps on Asian option pricing, we further consider the case of using normal distributed jumps. An analytical solution still exists under this setup but the asset value may fall below zero. We discuss this separately in Appendix A. Apart from normal distributed jumps, we explore the use of level dependent jumps in Appendix B, which also generates both upward and downward jumps. We do not specify the exact distribution of the jumps but will show that by imposing certain restriction on the jump distribution will lead us to an analytic solution of Asian options.

There are two reasons for using the CIR model as the base model. First, the CIR model is widely used in interest rate modeling and the characteristic function for $\int_0^T r_t\,dt$ exists. Here, we try to price the arithmetic Asian option, which involves the arithmetic average of asset prices resembling $\int_0^T r_t\,dt$. Second, by choosing suitable parameters satisfying the Feller condition, the CIR model allows the asset price to be modeled directly instead of the log asset price, while still maintaining the positivity of the asset value. Once the exponential distribution jump extension is introduced into the CIR model, the jump sizes are always positive and hence the positivity characteristic is maintained.

The setup of the problem is as follows. We are now standing at time 0 and try to price an option which matures at time T. As the goal is to price discretely monitored Asian options, we consider the underlying asset price being recorded at some regular time interval. Hence, the time horizon $[0,T]$ is split into $n+1\Delta$-spaced monitoring dates. These include $0, \Delta, \ldots, n\Delta = T$. Under this setup we are able to compute the analytic solutions for options with payoff depending on the terminal asset value $S_{n\Delta}$ and the arithmetic average value $Avg_n = \sum_{j=0}^{n}\alpha_j S_{j\Delta}$, where α_j is the weight put on $S_{j\Delta}$ such that $\sum_{j=0}^{n}\alpha_j = 1$. Notice that weights here do not necessarily need to be all equal. Table 1 shows the different types of options that can be priced analytically under this model.

2.2. Joint characteristic function

To obtain the analytical price of Asian options, we need the joint characteristic function for the pair $S_{n\Delta}$ and Avg_n. Therefore, we first

Table 1
Payoff functions for different options.

Option type	Payoff
European call	$\max\{S_{n\Delta} - K, 0\}$
European put	$\max\{K - S_{n\Delta}, 0\}$
Fixed strike Asian call	$\max\{Avg_n - K, 0\}$
Fixed strike Asian put	$\max\{K - Avg_n, 0\}$
Floating strike Asian call	$\max\{S_{n\Delta} - Avg_n - K, 0\}$
Floating strike Asian put	$\max\{K + Avg_n - S_{n\Delta}, 0\}$