Journal of Banking & Finance 37 (2013) 4353–4367

Contents lists available at ScienceDirect

Journal of Banking & Finance

journal homepage: www.elsevier.com/locate/jbf

Estimating the basis risk of index-linked hedging strategies using multivariate extreme value theory

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ARTICLE INFO

Article history:
Received 5 June 2012
Accepted 27 July 2013
Available online 8 August 2013

JEL classification:
C22
C51
G11
G32

Keywords:
Extreme value theory
Index-linked hedging instruments
Copulas

ABSTRACT

This paper studies the empirical quantification of basis risk in the context of index-linked hedging strategies. Basis risk refers to the risk of non-payment of the index-linked instrument, given that the hedger's loss exceeds some critical level. The quantification of such risk measures from empirical data can be done in various ways and requires special consideration of the dependence structure between the index and the company's losses as well as the estimation of the tails of a distribution. In this context, previous literature shows that extreme value theory can be superior to traditional methods with respect to estimating quantile risk measures such as the value at risk. Thus, the aim of this paper is to conduct an empirical analysis of basis risk using multivariate extreme value theory and extreme value copulas to estimate the underlying risk processes and their dependence structure in order to obtain a more adequate picture of basis risk associated with index-linked hedging strategies. Our results emphasize that the application of extreme value theory leads to better fits of the tails of the marginal distributions in the considered stock price sample and that traditional methods in regard to estimating marginal distributions tend to overestimate basis risk, while basis risk can in contrast be higher when taking into account extreme value copulas.

1. Introduction

Index-linked hedging strategies are of high relevance in finance and insurance and comprise derivatives such as futures or options as well as alternative risk management instruments such as cat bonds or industry loss warranties. One central problem associated with these risk management instruments is basis risk, which arises if the risk process underlying the hedge, e.g., a stock price index or a catastrophic loss index, and the hedging firm's position are not perfectly dependent. This implies a risk of non-payment if the index does not exceed the contractually defined (high) trigger level, even though the company suffers a critical loss.[3] This is not only critical from the perspective of the hedging firm, but also for regulators with respect to acknowledging these instruments as a risk transfer. There are two central aspects associated with the estimation of basis risk that require special attention, namely an adequate estimation of the tails of the marginal probability distribution function of the two risk processes (index and company's losses) and estimating the dependence structure between the two processes. In this context, previous literature has shown that extreme value theory (EVT) allows a better assessment of quantile risk measures as com-

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[3] As an example, consider a company that purchases a put option to hedge against high losses of its own firm value, which cannot be exercised (the put option is not triggered) since the put option's underlying does not suffer a loss that is high enough, i.e. the underlying does not fall below the strike price, given that the company's loss exceeds a critical loss level. Besides hedging financial market risk by means of financial derivatives, basis risk is also of high relevance in the context of risks related to insurance business operations, where risks of the insurer's underwriting operations are transferred by means of index-linked products. In this case, the buyer of the contract receives a payout, if an index (usually an industry, parametric or modeled loss index, see, e.g. SwissRe, 2009) exceeds a certain threshold. If this threshold is not exceeded, i.e. the industry loss for, e.g., a certain region in the U.S. or Europe is low, the contract does not provide the risk management measure needed, even though the buyer experiences a high loss from insurance contracts sold in the contractually defined region.

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http://dx.doi.org/10.1016/j.jbankfin.2013.07.043

pared to traditional approaches. Hence, the aim of this paper is to combine multivariate extreme value theory and extreme value copulas to estimate basis risk in order to obtain a more adequate picture of the risk associated with index-linked hedging strategies. This is done based on an empirical analysis, where we compare this method to traditional approaches.

In the literature, basis risk plays an important role for hedging strategies using financial derivatives such as futures or options, whose payments depend on changes in stock prices or indices (see, e.g. Figlewski, 1984; Moser and Helms, 1990; Castelino, 1992; Netz, 1996) as well as in the context of alternative risk transfer instruments in insurance markets (see, e.g. Major, 1999; Harrington and Niehaus, 1999; Cummins et al., 2004; Zeng, 2000; Gatzert and Kellner, 2011). Further important fields of interest with respect to basis risk are weather related risks (see Golden et al., 2007; Manfredo and Richards, 2009; Yang et al., 2009) and hedging strategies against price changes in the energy markets (see Haushalter, 2008). Basis risk is also of relevance when pricing hedging instruments (Wang and Wu, 2008; Lee and Yu, 2002).

However, when aiming to quantify basis risk or downside risk measures in general based on an empirical data sample, one problem is that the true probability distribution of the underlying risk process is usually unknown. Thus, traditional methods either use the empirical distribution function or estimate the whole marginal distribution based on the data sample. Even if these methods might be sufficient for most of the sample's observations, a drawback lies in the potential misestimation of the tails of the probability distribution and the non-consideration of extreme events such as catastrophic losses or a financial market crash. However, it is particularly the tails of the distribution, which are of relevance for most risk measures and in particular basis risk. An alternative to traditional approaches is the threshold exceedances method, which is based on extreme value theory and exclusively estimates the tail of a distribution, thus reducing potential misestimations in the tail. Moreover, the excess distribution in the tail converges to one of three possible distributions, whereas multiple distributions have to be estimated and compared if whole marginal distributions are considered. Furthermore, EVT and specifically the threshold exceedances method explicitly allow for the occurrence of extreme events, as heavy tails can be accounted for through the generalized Pareto distribution (GPD), which provides good estimates with respect to, e.g., the value at risk in the uni- and multivariate case and exhibits advantages as compared to traditional methods (see, e.g. Longin, 2000).

Due to these advantages, several papers apply extreme value theory to different fields of interest, including wind storm losses (see Rootzén and Tajvidi, 1997), loss distributions (see McNeil, 1997) or operational losses (see, e.g. Gourier et al., 2009). Extreme value theory is further used to examine effects arising from the dependence structure in the tails of multivariate distributions. While Zhou (2010) studies the impact of tail risks on diversification effects in a portfolio using the multivariate extreme value approach, Longin and Solnik (2001) and Poon et al. (2004) focus on the dependence structure among stock market indices and detect that common dependence models such as multivariate normality tend to underestimate tail dependencies, which might lead to an underestimation of the actual risk situation. In the field of insurance, Cébrian et al. (2003) price an excess of loss reinsurance contract whose payment depends on indemnity payments and allocated loss adjusted expenses, thereby taking into account the dependence structure through extreme value copulas. Further applications of multivariate extreme value theory to actuarial problems such as joint- and last-survivor annuities, hurricane losses in different regions or insurance portfolio composition are conducted by Dupuis and Jones (2006) as well as Brodin and Rootzén (2009).

Furthermore, the estimation of quantile risk measures by means of extreme value theory is subject to several analyses. Longin (2000) uses the block maxima method to estimate the value at risk of a long and short position in the S&P 500 and mixed portfolio positions, thereby using a linear model with respect to the dependence structure in the portfolio. This method is then compared to traditional approaches, i.e. the empirical and normal distribution as well as GARCH processes. The results suggest that the block maxima method might be better suited for the value at risk estimation, since the tails of the normal distribution may not be well fitted and GARCH processes might underestimate large unexpected market shocks. Hotta et al. (2008) compare the value at risk of a two asset portfolio using either the empirical distribution or a mixed distribution consisting of empirical distribution values and a GPD in the tail of the marginal distribution. They focus on two specific dependence models, logistic and asymmetric logistic dependence, and quantify the difference between these two approaches (with or without the use of the GPD) by means of backtesting without including the impact of different marginal distributions. A similar procedure is applied by Ghorbel and Trabelsi (2009) who estimate the value at risk of stock index portfolio positions and further include multivariate GARCH models in their comparison. With respect to their backtesting results, Hotta et al. (2008) and Ghorbel and Trabelsi (2009) both detect the extreme value approach to be superior to traditional methods.

In this paper, we contribute to previous literature by combining the two strands of the literature described above, the one on basis risk and the one on the application of extreme value theory in finance and insurance. Our aim is to show how basis risk can be estimated empirically in order to better assess the risk associated with index-linked hedging strategies. This is done by using multivariate extreme value theory and extreme value copulas. Toward this end, we conduct an empirical analysis using the S&P 500 as the index underlying the hedge and several firms listed in the S&P 500 as the assumed hedgers. We provide a formula for basis risk based on the estimated marginal distributions and dependence structure for the risk processes. Furthermore, in contrast to, e.g., Longin (2000), Hotta et al. (2008), and Ghorbel and Trabelsi (2009), we conduct a comparison of different estimation methods that is intended to provide an isolated analysis of the impact of potentially misestimating the tails of the marginal distributions and the dependence structure between these tails.

In particular, we first focus on marginal distributions and compare a traditional approach, which fits the marginal distributions of stock prices as a whole (based on the t and logistic distribution), with an EVT method that uses a GPD to estimate the probability values in the upper tail of the marginal distributions. Second, the impact of the dependence structure is taken into account with and without taking into account extreme value copulas. This procedure allows isolated insights regarding the effects of misestimating the marginal distribution and the dependence structure. Our results show that for all marginal distributions used in our analysis, the fit of the distribution can be improved if the data above the threshold is estimated through a GPD. Furthermore, for approximately half of our data, extreme value copulas capture the dependence structure in the joint tail distribution better than regular copulas that are estimated based on the whole distribution, and can thus be considered as a viable alternative. One major finding is that traditional methods regarding the marginal distributions tend to overestimate basis risk in the considered examples. In addition, the comparison between traditional and extreme value copula models emphasizes that the degree of asymptotic dependence is a key driver for basis risk. The remainder of this paper is structured as follows. Section 2 presents the theoretical background regarding extreme value theory and (extreme value) copulas as well as the