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Optimal smooth consumption and annuity design

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ABSTRACT

We propose an optimization criterion that yields extraordinary consumption smoothing compared to the well known results of the life-cycle model. Under this criterion we solve the related consumption and investment optimization problem faced by individuals with preferences for intertemporal stability in consumption. We find that the consumption and investment patterns demanded under the optimization criterion is in general offered as annuity benefits from products in the class of 'Formula Based Smoothed Investment-Linked Annuities'.

1. Introduction

This paper illustrates that preferences for smooth consumption streams can be explained by constrained marginal consumption and an associated quadratic optimization criterion. The optimal consumption stream derived under the preferences just mentioned, shows remarkably conformity with the benefit stream of a particular annuity product from the product class 'Formula Based Smoothed Investment-Linked Annuities'.

We were puzzled: Why are smooth-benefit and fixed annuities so much more popular than Unit-Link annuities with unsmoothed benefits?[1]

A wage earner's life can be divided into two phases; an accumulation and a decumulation phase. During the accumulation phase, the person consumes part of his earnings while saving the remainder for retirement. After the person has left the labor market, consumption is financed by decumulation of savings, typically through an annuity. The annuity is either bought in the accumulation period as a deferred annuity or at the date of retirement as an immediate annuity. Several types of annuities exists, e.g. fixed, with-profit, participating life, formula based smoothed investment-linked, Variable, Unit-Link annuity, etc. They differ in the stability over time in the benefits provided, ranging from a fixed annuity with a constant benefit stream to a pure Unit-link annuity where benefits vary perfectly with investment returns. In this paper we address the task of matching the preference structure of an individual to the consumption structure stemming from realized benefits of an annuity. Especially, we are interested in which preferences relate to the annuities providing the more smooth benefits.

The widely accepted "Life-Cycle Hypothesis" (LCH) of Modigliani & Broomberg and "Permanent Income Hypothesis" (PIH) of Friedman suggest that a person's consumption is proportional to his/her total wealth (the sum of financial and human wealth). In post-retirement, total wealth consists mainly of financial wealth

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[1] Insurance Information Institute (http://www.iii.org) reports $7.6 billions of individual immediate fixed annuities sales and $0.1 billions of variable annuities in 2010 in the US. The Association of British Insurers Research Department reports that new sales volumes in UK in 2007 consists of 89% conventional annuities (level/fixed and escalating) and 4% Unit Linked/With profit annuities, the rest being Enhanced/Impaired Life (see Gunawardena et al. (2008)).

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(human wealth is zero unless we account for public pension as part of human wealth). Consumption proportional to income thereby means that consumption in post-retirement varies perfectly with financial wealth, yielding that decumulation of savings is preferably done via a pure Unit-link annuity.

The fact that people do not decumulate via Unit-Link annuities is partly formulated in "The Consumption Smoothing Puzzle", which dates back to Hansen and Singleton (1983). They present evidence that observed consumption is much smoother than predicted by the life cycle models. Various solutions to the puzzle have been proposed, most of these modifying the market assumption underlying the model in Hansen and Singleton (1983), see e.g. Zeldes (1989) for stochastic income, Black (1990) for mean reverting returns or Fleming and Hernández-Hernández (2003) for stochastic volatility of returns.

Also preference modifications have been proposed in order to solve the puzzle. One preference modification that is of special interest here, is the introduction of endogenous habit formation, see e.g. Sundaresan (1989), Constantinides (1990) and Abel (1990). Furthermore, Munk (2008) solves the optimal investment and consumption problem with stochastic variations in investment opportunities and habit formation, thereby modifying both the underlying market and preferences. The key concept in habit forming preferences is intertemporal dependence in preferences in the sense that utility of present consumption depends on past consumption. Under these preferences, in comparison with the LCH and PIH, consumption is somewhat smoothed even for low or no human wealth.

This paper contributes to the understanding of the dependence structure in preferences that implies increased consumption smoothing. The optimization criterion proposed in this paper, though, does not include intertemporally dependent preferences but instead explicitly intertemporally dependent consumption. This is modeled by allowing for only limited control of consumption, in the sense that only the rate of change in the consumption rate is controllable.

In the object function of our proposed model we punish quadratic distance between the consumption rate derivative and a prespecified target and between terminal wealth and a target consumption ratio. The quadratic criterion is a classic in pension fund control where numerous papers examine the connection between the classical linear regulator and optimal pension funding. We refer the reader to the review article by Cairns (2000) and references therein. A main difference to our paper is that our criterion concerns the consumption rate derivative rather than the consumption rate itself. This adds a technical dimension through an additional state variable and changes the interpretation of the problem formulation and its solution. Also, we find that it makes sense to interpret the problem, not only as that of a pension fund controlling portfolio level contributions, but more as that of an individual decision maker, who expresses his preferences through a consumption growth rate target.

We find that the consumption patterns solving our proposed problem formulation shows remarkably conformity with the characteristics of annuity products from a particular class of products, namely "Formula Based Smoothed Investment-Linked Annuities". The conformity is established by direct comparison with one particular product from that product-class.[2]

The structure of the remainder of the paper is: Section 2 contains the classical results on optimal consumption and investment problems. In Section 3 we present and motivate our problem of optimal smooth consumption and we solve it in Section 4. Section 5 contains a formalization of the product "Tidspension" from the product-class "Formula Based Smoothed Investment-Linked Annuities". In Section 6 we compare the problem solution with the product characteristics of Tidspension.

[2] For a description of the product-class see Jorgensen and Linnemann (2012).

2. Classical results on consumption and investment

In this section we present the classical power utility continuous-time optimal investment and consumption problem and the related problem with habit persistence in preferences.

Academic literature on dynamic consumption and investment decisions in continuous-time starts with Merton (1969) and Samuelson (1969). The investment market in which the investor can invest consists of a bond with constant interest rate r and a stock with constant excess-return λ and volatility σ. Thereby, the investor faces the wealth dynamics

$$dX_t = (r + \pi_t\lambda)X_t dt + \pi_t X_t \sigma dW_t - c_t dt, X_0 > 0, \tag{2.1}$$

where W is a standard Brownian motion, π is the proportion of wealth invested in the stock and c is the consumption rate. The optimal strategies are derived for a time-additive power utility maximizer with constant relative risk aversion γ and time-preference coefficient ρ, facing the problem

$$\sup_{c,\pi}\mathbb{E}\left(\int_0^T \mathrm{e}^{-\rho s}\left[\frac{1}{1-\gamma}c_s^{1-\gamma}ds + \frac{1}{1-\gamma}X_s^{1-\gamma}d\varepsilon_T(s)\right]\right), \tag{2.2}$$

where $\varepsilon_T(\cdot) = \mathbb{1}_{\{T\leqslant\cdot\}}$.

The optimal solution for consumption (c^*) and investment proportion (π^*) is

$$c_t^* = \frac{X_t}{f_1(t)},$$
$$\pi_t^* = \frac{\lambda}{\sigma^2\gamma},$$

for a deterministic function f_1, see Merton (1969, 1971). The optimal consumption is proportional to savings which is very much in line with the LCH and the PIH. The investment proportion is constant and non-zero and we easily find that

$$dc_t^* = c_t^*(A_1(t)dt + \frac{\lambda}{\sigma^2\gamma}\sigma dW_t), \quad c_0^* = X_0/f(0),$$

for a deterministic function A_1. The point here is that consumption possess short term volatility in the sense that stock market fluctuations (through σdW) has an immediate effect on present consumption.

The related problem with additive endogenous habit formation in preferences consists of solving

$$\sup_{c,\pi}\mathbb{E}\left(\int_0^T \mathrm{e}^{-\rho s}\left[\frac{1}{1-\gamma}(c_s - h_s)^{1-\gamma}ds + \frac{1}{1-\gamma}(X_s - \xi h_s)^{1-\gamma}d\varepsilon_T(s)\right]\right),$$

subject to the wealth dynamics (2.1) and habit dynamics

$$dh_t = (\varphi_1 c_t - \varphi_2 h_t)dt, \quad h_0 > 0, \tag{2.3}$$

for $\xi \geqslant 0$ and $\varphi_1,\varphi_2 > 0$, see e.g. Munk (2008). With additive habit formation in preferences, utility from consuming c_t at time point $t \in [0,T]$ comes from the part of the consumption that exceeds a minimum consumption requirement given by the habit level h_t.

The optimal consumption and investment proportion for this problem is

$$c_t^* = h_t + (1 + \varphi_1 f_3(t))^{-\frac{1}{\gamma}}\frac{X_t - f_3(t)h_t}{f_2(t)},$$
$$\pi_t^* = \frac{\lambda}{\sigma^2\gamma}\frac{X_t - f_3(t)h_t}{X_t},$$