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Valuing catastrophe derivatives under limited diversification: A stochastic dominance approach

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ABSTRACT

We present a new approach to the pricing of catastrophe event (CAT) derivatives that does not assume a fully diversifiable event risk. Instead, we assume that the event occurrence and intensity affect the return of the market portfolio of an agent that trades in the event derivatives. Based on this approach, we derive values for a CAT option and a reinsurance contract on an insurer's assets using recent results from the option pricing literature. We show that the assumption of unsystematic event risk seriously underprices the CAT option. Last, we present numerical results for our derivatives using real data from hurricane landings in Florida.

1. Introduction

This paper examines the valuation of catastrophe instruments (CATs), financial contracts whose payoffs depend on the occurrence of a rare event that causes significant losses to a category of economic agents. A literature has been developed recently on the valuation of such contracts which, however, leaves many questions unanswered. We demonstrate here that the value of such instruments is crucially dependent on the assumptions made about the agents that would include such instruments in their portfolio. Plausible alternative assumptions may, in turn, bring major changes in the value of the instruments. We introduce the stochastic dominance methodology for valuing the catastrophe instruments and illustrate its application with numerical examples in the case of a European call option and a reinsurance contract.

Catastrophe insurance derivatives (futures and options) were introduced by the Chicago Board of Trade as early as 1992 as hedging instruments for the risk faced by insurers. They have not had much success as traded instruments in organized markets, although there is apparently active over-the-counter trading in them. As of mid-2010 the only such instruments listed in the Chicago Mercantile Exchange (CME) were futures and options contracts on the CHI, the CME Hurricane Index (formerly the Carville HI) for various parts of the US, but there was very little open interest and very few trades in the recent past. This does not eliminate the need for a valuation methodology for financial claims contingent on CAT events, but it does raise some questions about the assumptions on financial market equilibrium adopted during their valuation.

The importance of the valuation assumptions stems from the fact that in the presence of rare events financial markets are incomplete. Merton (1976), who was the first to note this property, suggested a contingent claims valuation method in which the rare event risk would be fully diversifiable and as such could be treated as unsystematic risk and not priced. The Merton assumption has been accepted by several authors valuing CAT financial instruments, who assume that there is an efficient reinsurance market that diversifies the CAT event risk,[2] but it is not to be accepted as a panacea, since it is clearly not applicable in many situations. As Duan and Yu (2005, p. 2441) note, catastrophe risk cannot be hedged if it has economy-wide implications.[3] More recently, Ibragimov et al. (2009) note that the efficient reinsurance assumption is not satisfied in real markets, since insurers specialize in geographical regions and

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[2] See, for instance, Dassios and Jang (2003), Duan and Yu (2005), Lee and Yu (2007), Lin and Wang (2009), and Chang et al. (2010).

[3] This was already known from the option pricing literature in the presence of event risk. See, for instance, Bates (1991).

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particular types of coverage, thus putting individual firms at high risk to specific catastrophe events.[4] These same authors show that catastrophe risk may be nondiversifiable even if it does not have economy-wide impact, depending on the characteristics of the probability distribution of the event risk.

In most of the literature the catastrophe event is modeled as a pure jump process, with Poisson arrivals and amplitudes that follow a given unspecified distribution.[5] Unfortunately the arbitrage methodology that is used almost exclusively in financial valuation is not particularly suited to the pricing of cash flows that depend on such rare events. This methodology uses most often continuous time valuation and a language that relies heavily on mathematical formulation, often ignoring troublesome elements of the underlying economic reasoning.[6] The difficulties are compounded by the fact that the underlying catastrophe process does not generally correspond to a traded financial instrument.

If catastrophe risk cannot be hedged then the dominant approach to the valuation of the rare event risk is to assume some kind of general equilibrium model that embodies strong assumptions like weak aggregation or the existence of a representative investor. This approach is used almost exclusively in the option pricing literature, and has only recently been introduced explicitly into the catastrophe instrument valuation.[7] The representative investor is almost always represented by a constant proportional risk aversion (CPRA) utility function, and the valuation results depend on the investor's risk parameter. This parameter enters into the transformation of the physical distribution of the occurrence of the catastrophe event (the *P*-distribution) into the risk neutral distribution used in the valuation of the financial instruments (the *Q*-distribution).

Apart from the fact that such a parameter is notoriously difficult to estimate and the values appearing in the literature vary from 0 to more than 50,[8] it is also completely unobservable in the case of most catastrophe instruments, which do not trade in organized financial markets. Most applications in finance estimate simultaneously the *P*-and *Q*-distributions, from the underlying security and the option market respectively. This presupposes liquid markets and sufficient alternative strike prices to generate the *Q*-distribution, which are found in stock index options but are unavailable in catastrophe derivatives.[9]

We solve this problem by adopting an alternative approach to the valuation of contingent claims, that of *stochastic dominance* (SD) which uses a much weaker set of assumptions than equilibrium. Unlike equilibrium, SD does not rely on the existence of a representative investor, let alone one with a CPRA utility function. Its only assumption is a pricing kernel that is monotone with respect to the contingent claim's payoffs. A sufficient condition for such monotonicity to be satisfied is the existence of a set of investors that hold portfolios comprised only of the underlying asset and other assets independent of it, as well as the riskless asset. The SD approach, originally introduced by Perrakis and Ryan (1984), Ritchken (1985), Levy (1985), Perrakis (1986, 1988), and Ritchken and Kuo (1988), has recently been extended to incorporate proportional transaction costs.[10]

The major advantage of SD in valuing CAT derivatives is the fact that the only information that it needs in order to value the contingent claim comes from the underlying asset's market, from the *P*-distribution. Instead of a single value for the contingent claim SD computes an upper and a lower bound on the value, which are reservation-purchase and reservation-write prices for the contingent claim. Violation of either one of these bounds allows the option holder to adopt a trading strategy that would increase the expected utility of *any* risk averse investor satisfying the conditions that guarantee a monotone pricing kernel. The bounds are derived in a discrete time multiperiod context, and are eventually extended to continuous time by a limiting argument.

We apply our methodology to the valuation of two contingent claims, both indexed on hurricane events. The first is a call option on hurricane intensity measured by the CHI, similar to the ones offered by the CME, while the second is a reinsurance contract on an insurer's assets. The two claims are different because the former is contingent on a pure jump while the latter on a jump diffusion process. The claim's value under the Merton (1976) assumption of fully diversifiable CAT event risk lies below the two bounds in the first case and coincides with the lower bound in the reinsurance contract. We also show, using realistic data from the CHI distribution that adopting the Merton (1976) assumption seriously underestimates the value of the CAT call option.

We elaborate on the model underlying the valuation of the CAT contingent claims in the next section and develop the multiperiod bounds in discrete time for the two claims that we study. Section 3 examines the convergence of the bounds to their continuous time limits. Section 4 presents some numerical results and compares SD to alternative valuation methods for CAT derivatives. Section 5 concludes.

In the remaining of this section we complete the literature review. As noted, almost all earlier studies adopt the Merton (1976) assumption that the CAT event risk is fully diversifiable. The differences in the valuation expressions come from alternative specifications of the continuous time dynamics of the CAT event and the associated financial claims on it. Geman and Yor (1997) and Muerman (2003) model the claim arrival process as a mixed jump-diffusion, in which the jump component has a fixed amplitude. Dassios and Jang (2003) use the Cox process to represent the amplitude of the CAT event. Duan and Yu (2005) use similarly jump-diffusion dynamics with stochastic interest rates and a lognormally distributed jump amplitude to model the contractual liability of an insurer facing catastrophe events. The stochastic interest rate feature is also present in the mixed jump-diffusion model of Jaimungal and Wang (2006). By contrast, Lee and Yu (2007) use a diffusion process with stochastic interest rates for the insurer's assets and liabilities and models separately the CAT event as a Poisson process with lognormal jump amplitudes, which they value using the Merton (1976) assumption. Lin and Wang (2009) use a jump diffusion model of asset dynamics to represent the aggregate catastrophe losses and apply it to value a perpetual American put option. Last, Chang et al. (2010) use a trinomial discrete time model to value the claim arrival process and a representative investor to evaluate the risk neutral distribution, but state, correctly, that "the introduction of utility functions to resolve the problems generated by the incomplete nature of the market is, in fact, often impractical as they are too much preference-specific"[11]: for this reason the authors use the Merton (1976) assumption in their numerical work. As this paper shows,

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[4] Similar remarks were also made by Barrieu and Loubergé (2009), who rely on behavioral considerations to account for the lack of diversification.

[5] See Geman and Yor (1997), Froot (2001), and Muerman, 2003.

[6] See, for instance, the discussion on market incompleteness in Geman and Yor (1997, p. 187) and in Bakshi and Madan (2002, pp. 107–108).

[7] See Chang et al. (2010), who use the representative investor in deriving the valuation expressions but assume that the event risk is fully diversifiable in their numerical work.

[8] See the survey article by Kocherlakota (1996).

[9] Muerman (2003) states that the link between the *P*-and *Q*-distributions can be found from simultaneously priced insurance derivatives and insurance contracts. Similarly, Chang et al. (2010) claim that it is possible to use observed reinsurance premiums in order to calibrate the prices of the catastrophe derivatives. These claims assume that markets are in equilibrium and that derivatives and underlying assets are "correctly" priced with respect to each other. As Constantinides et al. (2009, 2011) show, this is certainly not the case in S& P 500 index options and index futures options, which makes it highly unlikely that it would be true in CAT instruments.

[10] See Constantinides and Perrakis (2002, 2007). An empirical application is in Constantinides et al. (2011).

[11] See their footnote 6, p. 28.