Journal of Banking & Finance 37 (2013) 3286–3294

Contents lists available at SciVerse ScienceDirect

Journal of Banking & Finance

journal homepage: www.elsevier.com/locate/jbf

Multidimensional risk and risk dependence

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ARTICLE INFO

Article history:
Received 2 December 2011
Accepted 13 April 2013

JEL classification:
C52
C53

Keywords:
Multiple sources of risk
Multidimensional value at risk
Risk distribution
Dependence in risk
Systemic risk

ABSTRACT

Evaluating multiple sources of risk is an important problem with many applications in finance and economics. In practice this evaluation remains challenging. We propose a simple non-parametric framework with several economic and statistical applications. In an empirical study, we illustrate the flexibility of our technique by applying it to the evaluation of multidimensional density forecasts, multidimensional Value at Risk and dependence in risk.

1. Introduction

Most of the financial literature assumes that two considerations are of utmost importance for an investor: the reward that may be attainable and the inherent risk in obtaining this reward. The trade-off between reward and risk is the essence of any investment strategy. While it is straightforward to approximate the reward by the return on the investment, the definition of risk is more ambivalent since it involves quantifying various sources of uncertainty about the future investment value.

Conceptually, risk is the potential for (adverse) deviation from expected results. Different proxies for risk have been proposed, where perhaps the most popular in the univariate context is the variability of returns, as measured by the variance. If returns are not drawn from a normal distribution, then variance is no longer an appropriate measure of risk because it fails to capture some of the characteristics of the return distribution that investors consider important. An alternative univariate risk measure is the Value at Risk (VaR), which is defined as the maximum loss on a portfolio over a certain period of time that can be expected with a nominal probability. When returns are normally distributed, the VaR of a portfolio is a simple function of the variance of the portfolio (Szegö, 2002). However, when the return distribution is non-normal, as is now the general consensus, the VaR of a portfolio is determined not just by the portfolio variance but by the entire conditional density of returns, including skewness and kurtosis (see Tay and Wallis, 2000).

Risk management, generally, involves more than one risky asset and is particularly concerned with the evaluation and balancing of the impact of various risk factors. If the joint distribution of asset returns is multinormal, then the correlation coefficient adequately captures the dependence between assets (see Diebold et al., 1999). However, joint normality is not supported by empirical evidence (see, for example, Patton, 2004). Moreover, correlation is only a measure of linear dependence and suffers from a number of limitations (see Embrechts et al., 2002; Patton, 2004). These deficiencies are compounded in the covariance measure which is an explicit function of the individual variables variances and their correlation. The overreliance on covariances can have detrimental consequences as they are an essential input in many financial applications including hedging and portfolio decisions. Indeed, Embrechts et al. (2002) warn that unreliable risk management systems are being built using correlations – and by extension covariances – to model dependencies between highly non-normal risks.

While in the univariate context, the shortcomings of variance as a risk measure have been mostly addressed by VaR, the financial literature that explicitly addresses the shortcomings of the

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http://dx.doi.org/10.1016/j.jbankfin.2013.04.022

covariance as a measure of co-dependence is still in its infancy. Failure to properly characterize the relationships and inter-dependence of the multiple risk factors can have severe consequences as demonstrated by the recent failure of the rating agencies to account for house price risk when rating structured products (Gorton, 2010). Moreover, while the financial literature is replete with techniques which model the dependence in return (e.g. CAPM, APT), the equally important matter of the dependence in risk has only recently come to attention (see Patton, 2009).

This paper makes the following contributions to the nascent literature on multi-factor risk. Firstly, it proposes a simple and flexible statistical framework to evaluate time-varying, density forecasts of multidimensional risks. Secondly, VaR is generalized in a natural way. Essentially, multidimensional Value at Risk (MVaR) is a region of the intersection of univariate VaRs with a nominal probability mass under a given density function. It turns out that MVaR is a versatile framework that allows for examining and evaluating the dependence in risk. MVaR can also be seen as a straightforward illustration of the multiple sources of risk: If VaR is a univariate risk measure, which instead of the variance takes into account the entire tail density, then MVaR is a measure of multidimensional risk that instead of the covariances takes into account the entire distribution in the relevant joint tail.

The outline of the remainder of this paper is as follows. In Section 2, we present an economic motivation for MVaR, while in Section 3 we discuss the concept of joint density tails. In Section 4, we illustrate the application of this framework to multidimensional density forecasts (MDF) evaluation. Section 5 introduces MVaR and discusses its various statistical and economic interpretations, while Section 6 applies the MVaR framework to the measurement of the dependence in risk. Section 7 presents a small empirical study to illustrate these concepts. Finally, Section 8 concludes.

2. Motivation for multidimensional risk

Effective risk management requires not only correct identification of the sources of risk but also an adequate capturing of their distributional characteristics. Examples of the importance of properly accounting for the multiple sources of risk come from the financial economics literature. A major contribution to this literature, the Capital Asset Pricing Model (CAPM), models asset returns by decomposing their variability into market risk and firm-specific effects that can be diversified away in large portfolios. In this model, the return on the market portfolio summarizes the broad impact of macroeconomic factors. However, often rather than using a market proxy, it is more enlightening to focus directly on the ultimate, individual sources of risk. This can be useful in risk assessment, when measuring exposures to particular sources of uncertainty. Arbitrage Pricing Theory (APT) shows how a decomposition of risk into systematic and idiosyncratic influences can be extended to deal with the multifaceted nature of systematic risk. Multifactor models can be used to measure and manage exposure to each of the multiple economy-wide risk factors such as business-cycle risk, inflation, interest and exchange rate risk and energy price risk (see, for example, Ferson and Harvey, 1994; Chan et al., 1998).

The recent financial crisis brought to the forefront of attention systemic risk. This is the risk of collapse faced by the financial system as a whole when one of its constituent parts gets into financial distress. Due to the interconnectivity of the financial institutions, a shock faced by one institution in the form of an extreme event, increases the probability other financial institutions experiencing similar extreme events, leading to a domino effect (see Gai and Kappadia, 2010; Nijskens and Wagner, 2011). At the individual level, financial institutions are subject to three types of risk: market, credit and operational risk. For example, market risk typically generates portfolio value distributions that are often approximated as normal. Credit and especially operational risk generate more skewed distributions due to occasional extreme losses. For examples of market risk, see Jorion (2001). Crouhy et al. (2001) give examples of all three risk types while Kuritzkes et al. (2003) present stylized pictures of a very broad range of risk types that are faced by large financial companies. In recent years, there has been increasing concern among researchers, practitioners and regulators over the evaluation of models of financial risk. Moreover, while it is important to have an aggregate measure of the total risk, often it is also important to know the direct dependence on, and inter-relationships of, the specific market, credit and operational sources of risk. These developments accentuate the need for evaluation techniques that are flexible and yet powerful (Lopez and Saidenberg, 2000).

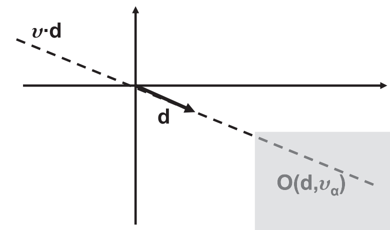

Fig. 1. Directed line $v \cdot d$ and a JDT $O(d, v_a)$ in R^2.

While the literature on aggregating the multiple sources of risk is recently gaining momentum (see, for example, Rosenberg and Schuermann, 2006), there appears to be virtually no research into the joint evaluation of such sources of risk or to characterize their inter-dependence. Moreover, while some risk types are more easily characterized and measured than others, much less is known about their joint behavior, distributional characteristics and cross-influences (see, for example, Chollete et al., 2011). By focusing on the joint distribution of the individual sources of risks, we provide a framework to characterize the co-dependence of these risks. It is important to emphasize that such a framework is not merely statistically interesting. As demonstrated by the recent financial crisis, financial institutions and regulators are in fact concerned with the possibility that their risk models do not adequately describe tail events. Indeed, a type of model failure of particular interest to financial institutions and regulators is that in which the forecasted probabilities of large losses are inaccurate or worse, underestimated.[1]

3. Joint density tails

In this section, we introduce definitions that we use throughout the paper. A joint density tail (JDT) is an unbounded region of the Euclidean space that is marked off by cut-off values. A parsimonious definition of the JDT $O(d, v)$ in the N-dimensional linear space R^N over the real line R requires only one cut-off value $v \in R$ and a directional vector $d \in R^N$ as illustrated in Fig. 1,

$$O(d, v) := \left\{ y \in R^N : y_i/d_i \geqslant v, \quad \forall d_i \neq 0 \right\} \tag{1}$$

[1] When the Federal Reserve Chairman Ben Bernanke was asked by the Financial Crisis Inquiry Commission what academic papers he recommends reading about the financial crisis and its aftermath, he suggested, among others, a paper by Adrian and Brunnermeier (2009), which proposes CoVaR ("Co" stands for conditional, contagion, or co-movement) as a way to measure a firm's systemic risk (see http://blogs.wsj.com/deals/2010/09/02/ben-bernankes-labor-day-reading-list/). CoVaR is a nested measure of our MVaR framework.