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# Speculations in option markets enhance allocation efficiency with heterogeneous beliefs and learning

Zhenjiang Qin *

*Institute of Financial Studies, Southwestern University of Finance and Economics, No. 55, Guanghuacun Street, Chengdu, Sichuan, 610074, China*

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ABSTRACT

Many studies investigate the impact of heterogeneous beliefs in the first moment, while very few in the second moment. This is partially due to continuous-time setup which makes it difficult to incorporate heterogeneous beliefs in the second moment. In a two-period exponential–normal model with Bayesian learning, I demonstrate that *heterogeneous prior variances* give rise to the economic value of option markets. Investors speculate in option market and public information improves allocation efficiency of markets only when there is heterogeneity in *prior variances*. Heterogeneity in mean is neither a necessary nor a sufficient condition for generating speculations in option markets. With heterogeneous beliefs, options are non-redundant assets which can facilitate side-betting and enable investors to take advantage of the disagreements and the differences in confidence. This fact leads to a higher growth rate in the investors' certainty equivalents and, thus, a higher equilibrium interest rate. Furthermore, option exhibits a unique feature of enabling signal precision to affect the *ex ante* risk premium of underlying asset, which quadratic derivative and stock do not have.



## 1. Introduction

A growing body of research has shown that disagreements in investors' beliefs on firm' fundamentals have significant impact on market characteristics including risk-free rate, risk premium, excess volatility, cross-section return, bond yield, and option pricing. Most of the literature focuses on the impact of heterogeneous beliefs in the first moment, while very few in the second moment.[1] This is partially due to continuous-time setup which makes it difficult to incorporate heterogeneous beliefs in the second moment. Since the Girsanov's Theorem implies that the instantaneous volatility of endowment is identical across investors under both individual perceived dynamics and risk-neutral dynamics. In a two-period model with an exponential–normal specification and Bayesian learning, this paper examines the impact of the heterogeneous prior beliefs in variance on market equilibrium in an incomplete market. The first major contribution in this paper is that *heterogeneous prior variances* provide economic value to option markets in the sense that the investors speculate in the option market and imperfect public signal improves the allocation efficiency of markets measured by investors' certainty equivalent only when there is heterogeneity in *prior variances*. Therefore, in my setting, the option is non-redundant. Whereas heterogeneity in mean is neither a necessary nor a sufficient condition for generating speculations in option markets.

On the other hand, public information or signal such as earnings and dividend announcements, mergers and acquisitions, macroeconomic announcements, accounting reports are long recognized to have substantial impacts on the financial markets.[2] However, also under an exponential–normal specification, both Brennan and Cao (1996) and Christensen and Qin (2013) show that when investors trade in stock and bond, and/or quadratic derivative, signal precision does not affect the *ex ante* risk premium of stock. Naturally, a question arises: Under what condition does public information quality exert influence on risk premium on stock and option? As the second main finding in the paper, I show that option payoff exhibits a unique feature of enabling signal precision to affect the *ex ante* risk premium of underlying asset, while nonlinear payoff of quadratic derivative and linear payoff of stock do not have this characteristic.

The model is a derivative oriented and two-period extension of the classic single-period capital asset pricing model (CAPM) with heterogeneous beliefs of Lintner (1969). Specifically, the investors

* Tel.: +86 28 87099045.
*E-mail address:* zqin@swufe.edu.cn

[1] Researches modelling heterogeneous beliefs in the first moment includes Detemple and Murthy (1994), Basak (2000), Basak (2005), and Buraschi and Jiltsov (2006), among many others.

[2] Recent empirical evidence suggests that macroeconomic announcements and employment figures have pronounced impact on financial markets. Literatures include, e.g., Feltham and Pae (2000), Andersen et al. (2003), and Richardson et al. (2005).

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hold different prior beliefs at $t = 0$ on the normally distributed $t = 2$ dividend, i.e., the prior beliefs of mean and precision (the inverse of variance) differ. These assumptions imply that the investors agree to disagree due to, for instance, difference in their experiences or DNA. The investors update their beliefs according to the Bayes' rule with a public signal received at $t = 1$ from a simple public information system.[3] The public signal is equal to the $t = 2$ dividend on the risky underlying asset plus independent noise. Moreover, the investors have concordant beliefs (Milgrom and Stokey, 1982) or homogeneous information beliefs (Hakansson et al., 1982) on the normally distributed noise in the signal, i.e., a zero mean and a common signal precision. These assumptions allow to measure the informativeness of public information system by the public signal precision. Furthermore, the investors can trade and speculate in the option markets, the underlying asset markets and the zero-coupon bond markets at $t = 0$ and $t = 1$, and consume at $t = 0$ and $t = 2$. Solving for equilibriums in an exchange economy, I investigate the impact of the heterogeneity in beliefs, the strike price, and the public information quality on risk-free rate, risk premium, and other asset pricing properties in option markets.

Heterogeneity in the prior variance creates the opportunities for speculation in option markets. With homogeneous prior variance, the investors do not trade in the option markets. The intuition is related to the results in Wilson (1968): Pareto efficient allocations in settings with heterogeneous beliefs require not only an efficient sharing of the risks, but also an efficient side-betting arrangement. With homogeneous prior variance, the Pareto efficient side-betting based on their disagreements about the mean can be achieved by trading only in the risky underlying asset and the zero-coupon bond at $t = 0$. The CAPM-like equilibrium price under heterogeneous beliefs is obtained. However, when the investors have different prior precision, trading only in the underlying asset and the zero-coupon bond at $t = 0$ does not facilitate efficient side-betting. The investors tend to speculate in the option markets. Take a European call option market for example, the investor with a low (high) prior precision takes long (short) position in the call option with convex payoff to achieve a terminal payoff which is a convex (concave) function of the dividend. This speculative strategy is the so-called Gamma trading strategy.[4]

Speculations in option markets increase the allocation efficiency of the equilibrium. This result can be detected from the change of asset pricing properties when investors' speculative behaviors change. First, conditional on identical average prior precision, the higher heterogeneity in beliefs, the more opportunities in speculations, and the more advantage of the disagreements and the differences in confidence among the investors can be taken. This effect leads to a higher efficiency of side-betting and more gains in trading options. The trading gains translate into increased certainty equivalents of the terminal consumption, and result in a higher equilibrium consumption growth, and thus a higher equilibrium interest rate. Second, investors tend to trade in options with an intermediate strike price. Since this type of options carry the most substantial convexity in their payoff, and thus the investors can effectively and actively speculate in the option markets. Third, the imperfect public signal facilitates speculations. When the investors have heterogeneous prior dividend precision, they update their posterior beliefs differently with imperfect public signal, and this gives the basis for additional trading gains contingent on the imperfect public signal. Another round of trading using Gamma trading strategies at $t = 1$ partly facilitates the efficient side-betting. Eventually, a combination of the option with intermediate strike price and public information system of the intermediate signal precision enables the investors to achieve the highest efficiency of side-betting, reflected by the unique maximum point of the *ex ante* equilibrium interest rate.

Options have a particular feature to allow public signal precision to affects the *ex ante* equilibrium risk premium on the risky underlying asset. The underlying mechanism is that the convexity of the option payoff at $t = 1$ varies with the variance of the posterior beliefs which are determined by the public signal precision. Through this relationship, the speculative positions in the underlying asset and the option are affected by the public signal precision. This fact gives rise to a signal-precision-dependent covariance between the marginal utility of consumption and the dividend and, thus, a signal-precision-dependent *ex ante* equilibrium risk premium. With an intermediate strike price, the impact of the public signal precision on the *ex ante* equilibrium risk premium is nontrivial (see Fig. 9). In contrast, also under an exponential–normal specification, both Brennan and Cao (1996) and Christensen and Qin (2013) show that when investors trade in stock and bond, and/or quadratic derivative, signal precision does not affect the risk premium. Note Brennan and Cao (1996) inject some unmodeled noise trading into the price system, while the noise trading does not bridge a relationship between the risk premium and the signal precision. On the contrary, speculations in option bridge this link.

### 1.1. Review of the literature

Some studies on the impact of information system and heterogeneity in beliefs on asset pricing are closely related to this work.[5] Back (1993) shows that when investors receive asymmetric information about the future price of an underlying asset, an introduction of option can cause a stochastic volatility of the underlying asset and make the option non-redundant. And Li (2012) assumes that investors believe the growth rate of the dividend to be a constant and known perfectly. This assumption enables closed-form solutions for vanilla European option prices and closed-form approximations for barrier options. His model offers a rationale for observed implied volatility patterns in an equilibrium setting and is also easy to implement in practice. However, in both models, the role of the option to facilitate side-betting is not explored, and both model are silent with respect to the influence of information system. Those issues are investigated in this paper.

In a similar effort, Buraschi and Jiltsov (2006) employ a model, based on the work by Detemple and Murthy (1994), to investigate the option markets with heterogeneous beliefs. They show that the heterogeneity in beliefs has significant pricing implications by plotting equilibrium asset pricing properties such as stock price and stock volatility as functions of the difference in beliefs. Their results indicate that the heterogeneous beliefs are strongly related to optimal portfolio holdings, stock volatility, equity premium, stock prices, option prices and skewness in equity returns.

My model differs with Buraschi and Jiltsov (2006) in several aspects. First, their studies focus on the impact of heterogeneity in means, while this paper mainly investigates the role of heterogeneity in prior variances. Second, Cuoco and He (1994) demonstrate

[3] Note this paper considers the economy-wide impacts of public information and, thus, the public information should be interpreted as, for instance, macroeconomic reports of aggregate consumption. And an information system is a set of potential signals that present conditional (or signal dependent) probabilities that each state at the terminal date occurs.

[4] See more about Gamma trading strategies in e.g., Hull, 2009, Chapter 17.

[5] Recent contributions of asset pricing in economies with incomplete information include, e.g., David (1997), Brennan (1998), Veronesi (1999) Veronesi (1999), Veronesi (2000), and Brennan and Xia (2001). Efforts to establish the so-called differences-of-opinion models in financial markets include, e.g., Cao and Ou-Yang (2009), and Banerjee and Kremer (2010). These literature assume the investors have homogeneous beliefs about the fundamentals in the economy, but disagree on how to interpret common public signals.