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Market capitalization and Value-at-Risk

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ABSTRACT

The potential of economic variables for financial risk measurement is an open field for research. This article studies the role of market capitalization in the estimation of Value-at-Risk (VaR). We test the performance of different VaR methodologies for portfolios with different market capitalization. We perform the analysis considering separately financial crisis periods and non-crisis periods. We find that VaR methods perform differently for portfolios with different market capitalization. For portfolios with stocks of different sizes we obtain better VaR estimates when taking market capitalization into account. We also find that it is important to consider crisis and non-crisis periods separately when estimating VaR across different sizes. This study provides evidence that market fundamentals are relevant for risk measurement.

1. Introduction

The accords on banking supervision from the Bank for International Settlements (BIS) single out Value-at-Risk (VaR) as a measure of financial risk. The BIS accords and in particular VaR play a central role in financial risk measurement and management. Despite its importance the most popular methods in practice for estimating VaR (historical simulation and RiskMetrics) are yet relatively simple. This is a constraint of real-world. The complexity of financial institutions calls for sound simple models, easy to estimate.

There is a vast academic literature on methods for estimating VaR. These methods can be very sophisticated and they are mainly reduced form in the sense that they explain risk in an autoregressive manner. The very well known GARCH model (Bollerslev (1986)) is perhaps the prime example of this. Recently there has been an increased interest on structural approaches for risk measurement involving market and macro-economic variables; see Andersen et al. (2012) for a detailed overview. This article contributes to this stream of research. Our aim is to understand the relation between stock size, measured by market capitalization, and equity risk measured by VaR.

By definition the $100\alpha\%$ VaR is the value such that the probability of observing a loss larger than VaR is smaller than the confidence level $1-\alpha$, over a given time horizon. The time horizon usually is a 1-day or 10-day holding period for market risk and 1 year for credit and operational risk. The confidence level α typically ranges between 95% and 99%. Hence VaR is in the tail of the profit-and-loss or returns distribution. This fact makes the estimation of VaR a difficult task. In probabilistic terms the definition of VaR is very simple. VaR is the negative of the $1-\alpha$ probability quantile of the returns distribution.

To compute VaR by the existing models it is necessary to obtain an estimate of the distribution of the portfolio returns sometime in the future. The only exception is the regression quantile method introduced in Chernozhukov and Umantsev (2001) and Engle and Manganelli (2004), where the quantile of the distribution is modeled directly. All the other VaR models use different approaches to estimating the distribution of the returns. We can classify these VaR models as follows: Historical simulation, introduced by Boudoukh et al. (1998), uses the empirical distribution function obtained from past data to estimate VaR as an empirical quantile; Filtered historical simulation estimates VaR as an empirical quantile of the residuals obtained from fitting a parametric model to the original returns. Most commonly the method is implemented with a GARCH type model to filter the returns as introduced by Barone-Adesi et al. (1998, 1999); Fully parametric methods which model the complete returns distribution. RiskMetrics (1996) and GARCH, from Bollerslev (1986), are prime examples of fully parametric models used for estimating VaR. Chavez-Demoulin et al. (2005) introduce a parametric sophisticated alternative to the use of

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GARCH as a filter based on a point process approach; Extreme Value Theory (EVT) methods model the tail of the returns distribution. The filtered EVT model, where the returns are filtered with a GARCH model, was first introduced in McNeil and Frey (2000).

Since the seminal work of Banz (1981), Stattman (1980), Rosenberg et al. (1985), and Fama and French (1992) that firm and market specific variables are known to be useful in explaining the expected return on stocks. Both the expected return and the quantile are characteristics of the asset return distribution. It seems pertinent to ask if there is a relationship between VaR, as a quantile, and market variables. Here we concentrate on market capitalization.

Reference papers comparing the performance of different VaR models are Bao et al. (2003, 2006), Brooks et al. (2005), Kuester et al. (2006) and Pritsker (1997). Many of the studies on the computation of VaR compare and propose different methods using data on large capitalization firms, major indices or highly traded currencies. A complete list would be long but relevant examples are: Kuester et al. (2006) who use daily returns on the NASDAQ Composite Index; McNeil and Frey (2000) do backtesting on the S&P500 and DAX indices, BMW stock prices, US dollar-British pound exchange rate and gold prices; Mancini and Trojani (2011) use the S&P500 index, US dollar-Japanese Yen exchange rate, Microsoft and Boeing stock prices; Bao et al. (2003) use daily returns on the S&P500 and NASDAQ indices; Engle and Manganelli (2004) implemented their CAViaR methodology on returns of General Motors, IBM and S&P500. Hence, there might be a bias in the results found in the literature concerning the performance of VaR estimation methods.

To the best of our knowledge market variables have not been often studied in connection with VaR estimation. The empirical study of VaR methods where we found market capitalization being used is Halbleib and Pohlmeier (2012). The authors raise the question whether market capitalization is important but the paper has a much wider focus exploring the performance of different VaR models and distributional assumptions across different estimation time windows. Although within a complex study, the authors find evidence that market capitalization is important for VaR estimation.

The contribution of this paper is to explore the importance of market capitalization in estimating VaR. We use returns on NYSE, AMEX and NASDAQ stocks. We specifically consider separately periods of financial crises and periods without crises, challenging the performance of methods for forecasting VaR. There is a number of methods for computing VaR. Since there is no ultimate consensus on which is the best we use several methods in our study. We calculate one period ahead (out-of-sample) sequences of VaR estimates. Then we compare the sequences of VaR estimates with the realized returns and test if the estimated VaR corresponds to the level of risk desired. The result of these tests gives the performance of the VaR methods. This is called backtesting a VaR model. We backtest VaR models on ten portfolios with different market capitalization during crisis periods and non-crisis periods. We test if the methods perform differently across different market capitalization portfolios during calm and during unstable market periods. Finally we estimate VaR for a portfolio composed of stocks of different sizes with and without taking market capitalization into account. Then we compare the performance of the two approaches to find if considering market capitalization significantly improves VaR estimation.

This article is organized as follows. In Section 2 we outline the methods for the estimation of VaR used in this study. Section 3 describes the backtesting methodology which quantifies the performance of the VaR methods. The empirical implementation of the VaR estimation methods and corresponding backtesting is reported in Section 4. In Section 5 we test the significance of market capitalization in explaining the performance of VaR methods for different stock size portfolios. We study the effectiveness of using market capitalization on estimating the VaR of a portfolio composed of stocks with different sizes in Section 6. Finally Section 7 synthesizes the results and presents final conclusions.

2. VaR estimation methods used in this study

The $100\alpha\%$ VaR is the negative of the quantile of probability $1-\alpha$ of the returns distribution. In most applications α varies between 95% and 99% but α can also take the value of 99.9% as for instance it is required for operational risk in the Basel II Accord. Formally, for a confidence level $\alpha \in (0,1)$, the $100\alpha\%$ VaR for period $t+h$, conditional on the information available up to time t, is given by

$$\mathrm{VaR}^{\alpha}_{t+h} = -Q_{1-\alpha}(R_{t+h}|\mathcal{F}_t) = -\inf\{r \in \mathbb{R} : P(R_{t+h} \leqslant r|\mathcal{F}_t) \geqslant 1-\alpha\}, \tag{1}$$

where R_t is the random variable representing the return in period t, $Q_\alpha(\cdot)$ denotes the quantile of probability α and $\mathcal{F}_t$ represents the information available at time t. Estimating VaR is equivalent to estimating a quantile of the unknown distribution of returns for period $t+h$.

The methods used here for estimating VaR can be classified as historical simulation (HS), fully parametric, and (semi-parametric) Extreme Value Theory (EVT) models. Historical simulation uses empirical quantiles obtained from (filtered or not) past data. Fully parametric models characterize the complete return distribution using a, possibly dynamic, parametric model. EVT models use a parametric family to describe the tail of the distribution while the center of the returns distribution is modeled by the empirical distribution function.

We assume that the returns can be defined as a location scale process conditional on the set of information available at time t:

$$r_{t+h} = E(R_{t+h}|\mathcal{F}_t) + \epsilon_{t+h} = \mu_{t+h} + \sigma_{t+h}z_{t+h}, \tag{2}$$

where μ_{t+h} is the expected return for the period $t+h$ given the information available at time t,σ_{t+h} is the conditional scale, ϵ_{t+h} is an error term and z_{t+h} has a zero location, unit scale probability density function $f_Z(\cdot)$. The $100\alpha\%$ VaR forecast for the period $t+h$ conditional on the information available at time t is then

$$\mathrm{VaR}^{\alpha}_{t+h} = -(\mu_{t+h} + \sigma_{t+h}Q_{1-\alpha}(Z)), \tag{3}$$

where Q_α is the α quantile of $f_Z(\cdot)$.

Different VaR methods assume different specifications for the conditional location μ_{t+h}, conditional scale σ_{t+h}, and probability density $f_Z(\cdot)$. An outline of the VaR methods used in this study follows.

2.1. Historical simulation

The simplest method of estimating VaR (see for instance Christoffersen (2012)) is to use the empirical quantile of the return distribution. This method is usually called (see Kuester et al. (2006)) the naive historical simulation. The theoretical justification for this estimator is that if we assume that the process of the returns is stationary then the empirical distribution is a consistent estimator of the unobserved future distribution function.

In order to define the estimator consider a sample of past ω returns $(r_t, r_{t-1}, \ldots, r_{t-w+1})$ and the ordered sample $(r_{(1)}, r_{(2)}, \ldots, r_{(\omega)})$, where $r_{(1)} \leqslant r_{(2)} \leqslant \cdots \leqslant r_{(\omega)}$ are the so-called ordered statistics. The historical simulation $100\alpha\%$ VaR for period $t+1$ is given by

$$\widehat{\mathrm{VaR}}^{\alpha}_{t+1} = -\widehat{Q}_{1-\alpha}(r_t, r_{t-1}, \ldots, r_{t-w+1}) = -r_{([(1-\alpha)\times\omega])}, \tag{4}$$

where $[\cdot]$ represents the integer part of a real number. As an example, if we consider a sample of return observations with size