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Robust portfolio choice with ambiguity and learning about return predictability

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ABSTRACT

We analyze the optimal stock-bond portfolio under both learning and ambiguity aversion. Stock returns are predictable by an observable and an unobservable predictor, and the investor has to learn about the latter. Furthermore, the investor is ambiguity-averse and has a preference for investment strategies that are robust to model misspecifications. We derive a closed-form solution for the optimal robust investment strategy. We find that both learning and ambiguity aversion impact the level and structure of the optimal stock investment. Suboptimal strategies resulting either from not learning or from not considering ambiguity can lead to economically significant losses.

1. Introduction

Numerous empirical studies conclude that excess stock returns are predictable in the sense that average excess stock returns depend on the current value of some predictor variable. The impact of return predictability on optimal dynamic portfolios has been studied in several settings. Some papers simply assume that the expected stock (index) return is an affine function of a given predictor variable, that the predictor follows a certain stochastic process, and that all parameters involved are known. However, parameters have to be estimated, and the assumed data-generating process might be misspecified. This paper studies how the optimal portfolio choice is affected by learning and ambiguity about return predictability and explores the interactions between learning and ambiguity.

We formulate a continuous-time model in which the expected excess stock return is the sum of an observable time-varying component, representing a known predictor, and an unobservable time-varying component. This captures the fact that any predictor is imperfect so that there are variations in expected stock returns beyond those caused by the chosen predictor. Since expected stock returns cannot be observed or estimated precisely, the second component is indeed unobservable. The investor can learn about the unobservable component from realized stock returns using Bayesian learning. Moreover, the investor is not sure about the filtered model and allows for some ambiguity. Being ambiguity-averse, he has a preference for investment strategies that are robust to misspecifications of the expected return in the reference model. The investor seeks an optimal robust investment strategy along the lines of Anderson et al. (2003) and Maenhout (2004). Our model thus exhibits both learning and ambiguity about return predictability.

Using Kalman filtering and maximum likelihood, we estimate the model based on monthly US return data from 1927 to 2010. We follow Boudoukh et al. (2007) and use the net payout yield (dividends plus equity repurchases less equity issuances) as our observable predictor. Compared to Boudoukh et al. (2007), we find that the net payout yield has a smaller predictive power since our model allows for another (unobservable) variable explaining variations in expected returns.

We derive the optimal robust investment strategy in closed form (numerical solution of simple ordinary differential equations is needed, though). We further derive closed-form expressions for the wealth-equivalent utility losses an investor suffers if he ignores learning about the unobservable predictor, or if he ignores the ambiguity about the model specification. Based on the estimated model, we illustrate key properties of the optimal strategy and the losses from the suboptimal strategies. Our results show that both learning and ambiguity have an impact on the size of the stock holdings and also on the structure of the optimal portfolio, with hedge terms for the uncertainty due to learning and due to

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ambiguity. Furthermore, investors with typical investment horizons and apparently plausible levels of risk aversion and ambiguity aversion will incur substantial wealth-equivalent utility losses if they ignore either learning or ambiguity when determining the investment strategy.[1] For a planning horizon of 20 years, the losses can easily exceed 50% of the initial wealth. The losses from ignoring to learn decrease in the risk aversion and in the ambiguity aversion of the investor since an increase in any of these preference parameters results in a more conservative strategy, and thus an error in the timing of the market is less severe. The loss from ignoring ambiguity naturally increases in ambiguity aversion, but decreases in risk aversion. Our results show that learning and ambiguity aversion are not substitutes. For an investor who is neither sure about the current value of the unobservable predictor nor about the true model, it is quantitatively important to take both learning and ambiguity into consideration when determining the investment strategy.

Next, we relate our paper to the existing literature. The returns on broad stock portfolios are reported to be predictable by such variables as the stock return in the recent past (Fama and French, 1988; Moskowitz et al., 2012); the price/dividend ratio, dividend yield, or net payout yield (Campbell and Shiller, 1988; Boudoukh et al., 2007); the price/earnings ratio (Campbell and Shiller, 1988); the book-to-market ratio (Kothari and Shanken, 1997); the short-term interest rate (Ang and Bekaert, 2007); the consumption-wealth ratio (Lettau and Ludvigson, 2001); the housing collateral ratio (Lustig and van Nieuwerburgh, 2005); the ratio of stock prices to GDP (Rangvid, 2006); and the variance-risk premium (Drechsler and Yaron, 2011). However, there are various statistical challenges in measuring predictability and there is still a lot of debate among academics about whether predictability is there or not (Ang and Bekaert, 2007; Cochrane, 2008; Lettau and Van Nieuwerburgh, 2008). Our modeling of predictability relates to that of van Binsbergen and Koijen (2010) who estimate a model with the (observable) dividend yield and the (unobservable) expected dividend growth rate as the predictors.

The impact of return predictability on optimal portfolio choice is derived and studied under the assumption of no parameter or model uncertainty by Kim and Omberg (1996), Campbell and Viceira (1999), and Wachter (2002), among others. Brennan (1998) investigates how learning about a constant expected return will affect the optimal portfolio. In a model with return predictability Barberis (2000) incorporates parameter uncertainty, but does not allow for dynamic learning. Xia (2001) assumes that the expected stock return is linearly related to a certain predictor and studies the optimal portfolio choice of an investor learning about the slope of this relation (where the slope is either constant or follows an Ornstein–Uhlenbeck process). Xia finds a substantial welfare cost of ignoring predictability or learning, in terms of a reduced certainty equivalent wealth. In her model, all variations in expected returns are due to the observable predictor, whereas we allow for additional variations via an unobservable predictor and also incorporate model uncertainty. Brandt et al. (2005) consider learning about other parameters of the return processes in addition to the predictive relation.

On the other hand, some papers investigate the effects on portfolio choice of an aversion against ambiguity about the return process. Ambiguity aversion can be modeled in various ways. We take the robust control approach suggested by Anderson et al. (2003). Maenhout (2004) adapts the idea to dynamic portfolio choice problems with constant relative risk aversion by imposing a homothetic specification of ambiguity aversion which renders the problem tractable and ensures that the optimal amounts invested in the different assets are proportional to wealth. He considers the simple Merton setting with a single stock and a risk-free asset with constant investment opportunities and assumes ambiguity about the expected rate of return on the stock. In an extension, Maenhout (2006) explores the role of ambiguity aversion when the expected stock return varies over time according to an Ornstein–Uhlenbeck process, as in the Kim and Omberg (1996) setting. Liu (2010) generalizes the analysis of Maenhout (2006) to Epstein-Zin preferences, whereas Liu (2011) considers a model with a regime-switching expected stock return with the current regime being unobservable.[2] We extend the model of Maenhout (2006) to the case where the expected stock return also has an unobservable component and the investor learns about this component based on observed stock returns and the observable component of the expected stock return. Our setting allows us to study the interactions between learning and ambiguity about stock return predictability.

The paper is organized as follows. Section 2 describes the model. Section 3 derives the optimal strategy with learning and ambiguity aversion and shows how to evaluate relevant suboptimal strategies. Section 4 estimates the model, discusses plausible levels of ambiguity aversion, and illustrates various aspects of the optimal investment strategies and the losses associated with strategies ignoring learning or ambiguity. Section 5 concludes. The appendices contain proofs and supplementary information.

2. Model setup

The investor has access to a risk-free asset (bond) with a constant rate of return r and a single risky stock. The stock price dynamics is described by the stochastic process[3]

$$dS_t = S_t[(r + a + b_x x_t + b_y y_t)dt + \sigma_S dz_{S,t}],$$

$$dx_t = \kappa_x(\mu_x - x_t)dt + \sigma_x dz_{x,t},$$

$$dy_t = \kappa_y(\mu_y - y_t)dt + \sigma_y dz_{y,t},$$

where x_t is an observable state variable, y_t is an unobservable state variable, and z_S, z_x, z_y are correlated one-dimensional standard Brownian motions under the reference probability measure $\mathbb{P}$. The expected excess return on the stock is thus given by

$$\mu_t = a + b_x x_t + b_y y_t,$$

that is the sum of a constant, an observable state variable, and an unobservable state variable. The observable state variable represents one of the known predictors mentioned in the introduction. In the numerical analysis in Section 4, the net payout yield plays the role of the observable predictor. The additional unobservable predictor y captures variations in the expected excess return beyond those caused by the chosen predictor. It reflects that any predictor (and also any set of predictors) is imperfect and will never explain all variations in the expected excess return.[4] The predictive power of the observable and unobservable state variable is given by the constants b_x and b_y, respectively.

If $b_y \neq 0$, the investor cannot observe the expected excess return but, from observing realized stock returns and the observable predictor, he can learn about the unobservable state variable using Bayesian learning. From Theorem 12.7 in Liptser and Shiryaev (2001), it follows that the filtered model (as seen by the investor) is given by

[1] Here, we base our identification of plausible levels of ambiguity aversion on the computation of detection-error probabilities as in Anderson et al. (2003), Maenhout (2006), and Liu (2010, 2011) for other portfolio choice settings.

[2] Pflug et al. (2012) study ambiguity in a Markowitz portfolio choice framework.

[3] We assume that the stock pays no dividends, however the analysis also holds for a dividend paying stock when the dividends are reinvested in the stock.

[4] A similar setup is used in van Binsbergen and Koijen (2010). Their model can be rewritten such that the expected return is driven by the (observable) dividend-price ratio and the (unobservable) expected dividend growth rate.