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Dynamic optimal portfolio choice in a jump-diffusion model with investment constraints

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ABSTRACT

We consider the dynamic portfolio choice problem in a jump-diffusion model, where an investor may face constraints on her portfolio weights: for instance, no-short-selling constraints. It is a daunting task to use standard numerical methods to solve a constrained portfolio choice problem, especially when there is a large number of state variables. By suitably embedding the constrained problem in an appropriate family of unconstrained ones, we provide some equivalent optimality conditions for the indirect value function and optimal portfolio weights. These results simplify and help to solve the constrained optimal portfolio choice problem in jump-diffusion models. Finally, we apply our theoretical results to several examples, to examine the impact of no-short-selling and/or no-borrowing constraints on the performance of optimal portfolio strategies.

1. Introduction

In this paper, we solve the optimal dynamic portfolio choice problem in a jump-diffusion model with some realistic constraints on portfolio weights, such as the no-short-selling constraint and the no-borrowing constraint. The dynamic portfolio choice problem without portfolio constraints in pure-diffusion models is prompted by the seminal work of Merton (1969, 1971) and Samuelson (1969), and is further developed by Karatzas et al. (1987), Kim and Omberg (1996), Wachter (2002), Detemple et al. (2003), and Liu (2007), among others. Wachter (2010) and Brandt (2010) are good references for portfolio choice problems. The constrained dynamic portfolio choice problem in pure-diffusion models is first studied by Karatzas et al. (1991), He and Pearson (1991), and Xu and Shreve (1992). In general, a market with portfolio constraints is incomplete. It is usually a daunting task to solve such a portfolio choice problem in an incomplete market, either through the HJB equation (due to limits on dimensionality) or the martingale-duality method (as there are infinitely many martingale measures). To overcome the market incompleteness caused by portfolio constraints, Cvitanic and Karatzas (1992) propose a general approach to solve dynamic portfolio choice in the presence of various constraints on portfolio weights, including no-short-selling constraints and no-borrowing constraints.

More precisely, by appropriately adjusting the risk-free rate and the drift terms of risky asset prices, Cvitanic and Karatzas (1992) convert the constrained portfolio choice problem in the original incomplete market into an unconstrained one in a set of fictitious complete markets. Hence, solving the optimal portfolio problem in the original incomplete market can be reduced to that in a set of fictitious complete markets. As a result, we can apply the standard martingale method to solve the optimal portfolio problem in each fictitious complete market. Furthermore, it has been shown that the optimal consumption and portfolio rule in the original market is identical to those which are optimal in the worst of all the fictitious markets. However, it is generally hard to find the worst fictitious market and the corresponding optimal consumption and portfolio strategy in the presence of a large number of state variables. For this reason, Bick et al. (2013) have recently developed some efficient simulation-based approximation algorithms to solve constrained consumption-investment problems in pure-diffusion markets via the martingale-duality approach.

In those models mentioned above, it is standard to assume that asset prices follow pure-diffusion processes, primarily due to their analytical tractability. However, much recent research in finance has documented empirical evidence of jumps in stock returns. See, for example, Bakshi et al. (1997), Bates (2000), and Eraker et al. (2003). With jumps, an asset return model can explicitly allow for sudden but infrequent market movements of large magni-

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tude, and thus capture the "skewed" and "fat-tailed" features of stock return distributions. Many empirical and theoretical studies find that the jump risk has a substantial impact on portfolio choice, risk management and option pricing. See Merton (1976), Liu et al. (2003), and Duffie et al. (2000), for example. More specifically, recent portfolio choice papers in jump-diffusion models demonstrate that optimal portfolios held by an investor facing jump risks differ markedly from those in the absence of jumps, and that the economic loss of ignoring jumps may be substantial. For a more detailed analysis, see Liu et al. (2003) and Das and Uppal (2004).

Given the substantial impact of jumps on an investor's asset allocation decision, this paper solves the optimal portfolio choice problem in realistic settings which involve jumps in stock returns, portfolio constraints and potentially a large number of state variables. As demonstrated by Bardhan and Chao (1996), once unpredictable jumps are incorporated, a model with or without portfolio constraints is inherently incomplete, regardless of the number of traded stocks. This is in striking contrast to pure-diffusion models which can be completed by incorporating more stocks. Hence, unlike a pure-diffusion model with portfolio constraints, the incompleteness caused by jumps in a jump-diffusion model can not be removed through the "fictitious completion" techniques in Karatzas et al. (1991) and Cvitanic and Karatzas (1992) and thus, the martingale duality approaches they used cannot be directly applied to a jump-diffusion model. In this paper, we solve the optimal portfolio choice problem in a multi-asset jump-diffusion model with portfolio constraints. To be more specific, we first establish equivalent optimality conditions similar to those in Cvitanic and Karatzas (1992), which convert the constrained portfolio choice problem in the original jump-diffusion model into an unconstrained one in a set of fictitious jump-diffusion models. Then, we apply a portfolio weight decomposition approach recently developed by Jin and Zhang (2012) to solve the portfolio choice problem in jump-diffusion models.

Our paper is related to several papers in the literature on portfolio choice problems in a jump-diffusion setting. The model in the present paper, however, differs from those used by Liu et al. (2003) who consider single-stock jump-diffusion models with no portfolio constraints, while we consider multi-asset jump-diffusion models with some realistic portfolio constraints. In Das and Uppal (2004) and Ait-Sahalia et al. (2009), meanwhile, they solve the portfolio selection problems in jump-diffusion models which can include a large number of assets. However, in their models, there is only one type of jumps. All of the coefficients in stock return processes are constants and there are no portfolio constraints. In contrast, we consider the optimal portfolio strategies in a multi-asset jump-diffusion model, which includes a large number of assets, state variables and portfolio constraints.

Our paper is also related to Jin and Zhang (2012) on portfolio choice problems in a jump-diffusion setting, in which the authors develop decomposition methods for portfolio weights to obtain tractable solutions to optimal portfolio strategies in a jump-diffusion model incorporating a large number of assets and state variables. However, only one portfolio constraint is considered. The constraint is that the number of traded risky assets is smaller than the total number of diffusions and jumps, which is the case of an incomplete pure-diffusion market considered in Karatzas et al. (1991). In the present paper, we incorporate more general constraints in a jump-diffusion model.

In short, our work contributes to the dynamic portfolio choice literature by extending the pure-diffusion model in Cvitanic and Karatzas (1992) to a jump-diffusion model and incorporating more general portfolio constraints in Jin and Zhang (2012). To the best of our knowledge, our paper is the first one to consider general and realistic constrained portfolio choice problems in jump-diffusion models with a large number of assets and state variables.

The rest of the paper is organized as follows. In the next section, we lay out the framework of the constrained dynamic portfolio choice problem in a jump-diffusion model, construct an unconstrained dynamic portfolio choice problem in a set of fictitious markets, and then present our results of equivalent optimality conditions. Section 3 applies the theoretical results developed in Section 2 to no-short-selling and no-borrowing constraints respectively, and compares the method in the present paper with the standard HJB equation method. Section 4 applies the theoretical results to several numerical examples. Section 5 concludes the paper. All proofs are given in the appendices.

2. The portfolio choice problem

This section describes the investment problem for an investor in allocating her wealth between a set of risky assets and one risk-less asset in a jump-diffusion model, who faces investment constraints. The investor seeks to maximize the expected utility from intermediate consumption and terminal wealth.

2.1. The constrained portfolio choice problem

We fix a complete probability space $(\Omega, \mathcal{F}, P)$ and a filtration $\{\mathcal{F}_t\}$ satisfying the usual conditions. In the economy assumed, we use a l-dimensional state variable $X_t = (X_{1,t}, \ldots, X_{l,t})^\top$ to capture the stochastic variation in investment opportunities. Stochastic volatility and interest rates are typical examples of state variables. Here we use $\top$ to denote the transpose of a matrix or a vector. For analytical tractability, as illustrated in Jin and Zhang (2012), we assume that state variables X_t follow a pure diffusion process

$$dX_t = b^x(X_t)dt + \sigma^x(X_t)dB_t^X$$

where $B_t^X = \left(B_{1,t}^X, \ldots, B_{l,t}^X\right)^\top$ is an l -dimensional standard Brownian motion, $b^x(X_t)$ is an l-dimensional vector function and $\sigma^x(X_t)$ is an $l \times l$ matrix function of X_t.

An investor with a planning horizon $[0,T]$ seeks to allocate her wealth between one risk-less asset and n risky securities with portfolio constraints described below. The risk-less asset, called the bond, has a price $S_{0,t}$ which evolves according to the differential equation

$$dS_{0,t} = S_{0,t}r(X_t)dt \qquad (1)$$
$$S_{0,0} = 1$$

The prices of risky assets follow the dynamics

$$dS_{i,t} = S_{i,t-}\left[b_i(X_t)dt + \sigma_i^b(X_t)dB_t^S + \sigma_i^q(X_t)(Y \bullet dN_t)\right] \quad \text{for } i = 1, \ldots, n \qquad (2)$$

where $B_t^S = \left(B_{1,t}^S, \ldots, B_{d,t}^S\right)^\top$ is a d-dimensional standard Brownian motion correlated with B_t^X with a $d \times l$ correlation matrix ρ_t, and $N_t = (N_{1,t}, \ldots, N_{n-d,t})^\top$ is a $(n-d)$-dimensional multivariate Poisson process, with $N_{k,t}$ denoting the number of type k jumps up to time t. $\sigma_i^b(X_t)$ is the d-dimensional diffusion coefficient vector and $\sigma_i^q(X_t)$ is the $(n-d)$-dimensional jump coefficient vector. $Y = (Y_1, \ldots, Y_{n-d})^\top$ is a $(n-d)$-dimensional vector and $Y \bullet dN_t$ denotes the component-wise multiplication of Y and dN_t. More precisely, $Y \bullet dN_t = (Y_1 \bullet dN_t = (Y_1 dN_{1,t}, \ldots, Y_{n-d}dN_{n-d,t})^\top$, where Y_k denotes the size of the type k jump. In particular, the Brownian motions represent frequent small movements in stock prices, while the jump processes represent infrequent large shocks to the market.

For illustrative purposes, we assume that N_k has finite activity with stochastic intensity λ_k, and the size Y_k of the type k jump has probability density $\Phi_k(t, dx)$.[1] For tractability, we assume λ_k = -

[1] Our results can be extended to a model with infinite activity by replacing $\lambda_k(t)\Phi_k(t, dx)$ with a Levy measure.