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Forecasting liquidity-adjusted intraday Value-at-Risk with vine copulas

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We propose to model the joint distribution of bid-ask spreads and log returns of a stock portfolio by using Autoregressive Conditional Double Poisson and GARCH processes for the marginals and vine copulas for the dependence structure. By estimating the joint multivariate distribution of both returns and bid-ask spreads from intraday data, we incorporate the measurement of commonalities in liquidity and comovements of stocks and bid-ask spreads into the forecasting of three types of liquidity-adjusted intraday Value-at-Risk (L-IVaR). In a preliminary analysis, we document strong extreme comovements in liquidity and strong tail dependence between bid-ask spreads and log returns across the firms in our sample thus motivating our use of a vine copula model. Furthermore, the backtesting results for the L-IVaR of a portfolio consisting of five stocks listed on the NASDAQ show that the proposed models perform well in forecasting liquidity-adjusted intraday portfolio profits and losses.

1. Introduction

Liquidity risk is of major concern to both investors and portfolio managers. Especially in times of market turmoil, overall market liquidity can dry up forcing investors to exit positions at increased cost. Consequently, investors prefer assets which are either liquid (see Amihud and Mendelson, 1986; Brennan and Subrahmanyam, 1996; Liu, 2006) or are least not exposed to systematic decreases in liquidity (Acharya and Pedersen, 2005; Sadka, 2006). While the finance literature has long been concerned with liquidity risk and market microstructure (Demsetz, 1968; Stoll, 1978; Amihud and Mendelson, 1980), the recent financial crisis has renewed interest in both the modeling of liquidity risk and in the analysis of its driving factors (see e.g. Cornett et al., 2011; Dick-Nielsen et al., 2012).

In this paper, we propose a multivariate econometric model based on vine copulas for estimating and forecasting liquidity-adjusted risk measures for multivariate stock portfolios and apply the proposed model on returns and bid-ask spreads reconstructed from high frequency data. We start our analysis by performing a variety of diagnostic tests on the dependence structure between intraday stock returns and quoted bid-ask spreads for selected companies on the NASDAQ 100 in 2009.[2] These preliminary tests provide us with ample empirical evidence of not only strong linear correlation but also strong tail dependence between bid-ask spreads across individual stocks. We then turn to modeling the marginal behavior of stock returns and bid-ask spreads by the use of GARCH processes and the Autoregressive Conditional Double Poisson model, respectively (Heinen, 2003; Groß-Klußmann and Hautsch, 2011). As the dependence structure between intraday stock returns and bid-ask spreads stemming from different companies has not been analyzed before in the literature,[3] our dependence model is required to be able to capture a wide range of possible linear and non-linear dependencies. We therefore resort to the concept of vine copulas (Heinen and Valdesogo, 2008; Aas et al., 2009) to model the dependence structure between the returns and bid-ask spreads of multiple stocks. Based on our estimated multivariate model for returns and bid-ask spreads, we then forecast and backtest several types of liquidity-adjusted risk measures to illustrate the superiority of our proposed method.

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[2] Obviously, other measures of liquidity could have also been used. The majority of studies in the risk management literature, however, regularly employs the quoted bid-ask spread to measure liquidity (see e.g. Berkowitz, 2000; Bangia et al., 2002).

[3] The relationship between the returns and the bid-ask spread of the same company, on the other hand, is well documented, see e.g. Amihud and Mendelson (1986) who find that average portfolio risk-adjusted returns increase with their bid-ask spread with the slope of the return-spread relationship decreasing with the spread.

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Although early empirical and theoretical studies on liquidity have concentrated on individual securities, the analysis of comovements in liquidity among individual stocks has become a corner stone of the microstructure literature starting with the seminal papers by Chordia et al. (2000), Hasbrouck and Seppi (2001) and Huberman and Halka (2001). Since then, studies on the commonality in liquidity have unanimously found clear empirical evidence for strong comovements in liquidity as proxied, e.g., by the bid-ask spreads of individual stocks. The determinants driving this commonality in liquidity, however, have remained relatively unknown until the recent study by Karolyi et al. (2012). They find that commonality in liquidity is positively correlated with high market volatility. In addition, liquidity commonalities were particularly strong during the Asian crisis and the recent financial crisis underlining the necessity of taking liquidity risk into account in multivariate risk modeling.

The incorporation of liquidity risk into the measurement of market risk has been a recurring topic in the risk management literature since the onset of the Value-at-Risk (VaR) concept as a de facto industry standard. Although standard VaR lacks a rigorous consideration of liquidity risk, several extensions to certain forms of Liquidity-Adjusted VaRs (L-VaR) have been proposed in the literature (see e.g. Berkowitz, 2000; Bangia et al., 2002; Qi and Ng, 2009). Parallel to the study of liquidity-adjusted risk measures, the availability of tick-by-tick data has enabled researchers to measure the liquidity of stocks at an ultra-high frequency (Chordia et al., 2000; Engle, 2000). While the increase in the speed of trading and the widespread availability of transaction data have led to an increase in the importance of intraday risk analysis (Dionne et al., 2009; Gourieroux and Jasiak, 2010), the modeling of intraday stock returns and bid-ask spreads is severely hampered by the irregular spacing of such data. Econometric research has thus concentrated on deriving appropriate models for the duration between transactions (Engle and Russell, 1998), for the estimation of intraday volatility (Andersen et al., 2000, 2003) and for the estimation of intraday risk measures (Giot, 2005; Dionne et al., 2009).

The econometrics literature includes several studies which focus on modeling and forecasting multivariate intraday stock returns or multivariate bid-ask spreads (see e.g. Engle and Russell, 1998; Breymann et al., 2003; Heinen and Rengifo, 2007; Groß-Klußmann and Hautsch, 2011; Li and Poon, 2011). Up to date, however, no study has analyzed the joint distribution of intraday returns and bid-ask spreads of multiple stocks. Although stock returns are known to be leptokurtic and non-normally distributed with time-varying dependence (see e.g. Longin and Solnik, 1995), and bid-ask spreads are known to comove across markets, little is known about the dependence (and consequently possible comovements) between liquidity and returns of different stocks.

In addition, the multivariate modeling of bid-ask spreads has so far been a purely econometric exercise with no connection to the rich literature on liquidity commonality.[4] Furthermore, there exist only few studies on the estimation of VaR from intraday data (Dionne et al., 2009) and, to the best knowledge of the authors, no work on multivariate liquidity-adjusted portfolio-VaR. Our paper tries to link the research on liquidity commonality and on the estimation of multivariate portfolio-VaR from high frequency data.

The contributions of this study relative to the existing literature on liquidity commonality and liquidity risk management are significant and numerous. While previous studies on the commonality in liquidity have documented strong positive *linear correlation* between measures of liquidity, this study is the first to find empirical evidence for a strong *non-linear* dependence in the form of significant tail dependence as well. This paper is also the first to integrate our understanding of liquidity commonality into the estimation and forecasting of liquidity-adjusted risk measures. In addition, this paper constitutes the first multivariate model for the joint distribution of the returns and bid-ask spreads of multiple stocks. Finally, our paper adds to the fastly growing literature on the use of vine copulas in risk management and asset pricing applications making full use of the vines' flexibility.

The results presented in this paper show that the proposed multivariate model for bid-ask spreads and intraday returns performs exceptionally well in forecasting liquidity-adjusted portfolio losses. While losses are adequately bounded below by our liquidity-adjusted VaR forecasts, our models are not too conservative and thus prevent investors like, e.g., banks from reserving unnecessary risk capital buffers. At the same time, we show that the neglection of liquidity risk and non-linearities in the dependence between returns and bid-ask spreads in the forecasting of portfolio-VaRs can lead to a severe underestimation of losses on the portfolio.

The remainder of this article is structured as follows. Section 2 presents the econometric methodology. Section 3 discusses the empirical study as well as the results. Concluding remarks are given in Section 4.

2. Econometric methodology

The purpose of this section is to outline the econometric models for intraday bid-ask spreads and returns, their multivariate dependence structure as well as liquidity-adjusted intraday VaR.

We start with the models for intraday spreads and returns which form the fundamental building blocks of the VaR models described later.

2.1. The autoregressive conditional double poisson model

Loosely speaking, liquidity can be seen as the ability of a market to allow for immediate trading even large amounts to minimal costs without causing remarkable price movements; hence, liquidity risk results from the difference between transaction and market price.

Kyle (1985) provides a formal definition and introduces three components of liquidity including tightness, depth and resiliency; according to Amihud and Mendelson (1986), a natural measure of liquidity (or rather liquidity risk) is the spread between the bid and ask prices. Bangia et al. (2002) classify liquidity risk into exogenous liquidity risk which arises from the general conditions of a market and is equal to all participants and endogenous liquidity risk which refers to the volume of individual trading positions and which is idiosyncratic.

The bid-ask spread has become a key parameter in modeling financial data and captures the costs of immediate trading, which can be explained by the liquidity suppliers' purchasing at the bid and selling at a higher ask price in order to recoup their own costs. Empirical properties like intraday seasonalities, linear dependence and commonalities are given in the context of the data description in Section 3.

Statistically, bid-ask spreads belong to the class of discrete count data, because they count the number of ticks between bid and ask prices. To accurately model time series of count data measured at high frequency, there are two important aspects to consider: on the one hand the model has to cope with the issues of discreteness and serial dependence, on the other hand its estimation procedure has to be tractable for a large number of observations. Following Groß-Klußmann and Hautsch (2011), we

[4] As stated by Heinen and Rengifo (2007), extending the standard Autogressive Conditional Duration (ACD) model of Engle and Russell (1998) to more than one time series has proven to be quite difficult. Considering the numerous problems one encounters when trying to model multivariate count data, it is, however, not surprising that studies such as the one by Heinen and Rengifo (2007) focus on the econometric side of the problem.