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The structure and degree of dependence: A quantile regression approach

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ABSTRACT

The copula function defines the degree of dependence and the structure of dependence. This paper proposes an alternative framework to decompose the dependence using quantile regression. We demonstrate that the methodology provides a detailed picture of dependence including asymmetric and non-linear relationships. In addition, changes in the degree or structure of dependence can be modeled and tested for each quantile of the distribution. The empirical part applies the framework to three different sets of financial time-series and demonstrates substantial differences in dependence patterns among asset classes and through time. The analysis of 54 global equity markets shows that detailed information about the structure of dependence is crucial to adequately assess the benefits of diversification in normal times and crisis times.

1. Introduction

> *Dependence among random variables is a nasty business and we usually ignore the nastiness (at our peril) by employing measures of linear association like the Pearson correlation coefficient.* (Koenker, 2002)

The above statement identifies an incomplete characterization of dependence among random variables with the linear correlation coefficient. The global financial crisis has highlighted that markets co-move not only on average but particularly during crises often exacerbating an initial shock and eliminating the expected beneficial effects of diversification. In addition, changes of (inter-) dependence in a crisis often show that the dependence of assets or markets is non-linear and asymmetric. There is a large literature on the modeling of dependence. Studies without a specific focus on crises periods, inter alia, are Ang and Chen (2002), Cappiello et al. (2006), Garcia and Tsafack (2011), Longin and Solnik (2001) and Poon et al. (2004). Embrechts et al. (2002) discuss the properties and pitfalls in dependence modeling and Embrechts et al. (2003) and Cherubini et al. (2004) describe the modeling of dependence with copulas.[1] Specific applications of copulas to risk management and diversification of financial risks can be found in Bhatti and Nguyen (2012), Chollete et al. (2011) and Kole et al. (2007). The use of quantile regression to model the dependence of financial variables, e.g. trading volume and return volatility, is introduced by Chuang et al. (2009) and Lee and Li (2012).

The literature which concentrates on changes of financial market dependencies in a crisis period compared to a normal period has grown substantially in recent years and is commonly related to the term *contagion*. Baig and Goldfajn (1999) and Forbes and Rigobon (2002) use a linear dependence measure, Bae et al. (2003) and Baur and Schulze (2005) use coexceedances estimated within a multinomial logit framework and a quantile regression framework, respectively. Boyer et al. (2006) use a Markov-switching model to estimate changes in dependence and Hu (2006) uses copulas. Rodriguez (2007) finally combines the Markov-switching model and copulas to estimate changes of dependence in crisis periods relative to tranquil periods.

Hu (2006) introduces the terms *degree* and *structure* of dependence in the copula framework and thus emphasizes that financial turmoil can induce changes in dependence along two dimensions, i.e. an episode of financial turmoil can change the degree of dependence and the structure of dependence.

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[1] See also Patton (2012) for a recent review of copula models for economic time series.

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This paper takes the copula-based decomposition into two components as a starting point to introduce an alternative, semi-parametric, framework. We show that quantile regression provides a flexible modeling and estimation method to identify the degree and structure of the dependence of two financial assets.

We contribute to the literature in two major ways. First, we propose a novel methodology to isolate two components of dependence, that is, we use quantile regression to estimate the degree and the structure of dependence. The framework is related to the theory of copulas but differs in several respects. For example, the dependence of two variables is estimated for 99 quantiles without an ad hoc selection of a certain structure of the dependence. In comparison to the copula framework, there is no need to estimate different copulas or mixtures of copulas and select the one with the best fit. In addition, the quantile regression model circumvents the commonly employed two-stage estimation strategy for copulas: there is no need to separately estimate the marginal distributions and to condition on certain variables in a first stage to obtain the conditional dependence. The conditional dependence can be estimated directly. Furthermore, the statistical significance of the dependence and the conditioning variables is provided for each quantile of the distribution.

The second contribution is empirical in nature and shows that the degree and structure of dependence between asset returns changes through time for some asset classes. Three different data sets are used as an empirical application of the framework: (i) the study of the DM (Euro)-US dollar and Yen-US dollar exchange rates before and after the introduction of the euro illustrates that the euro-introduction led to a lower degree of dependence with negligible changes in the structure, (ii) the analysis of gold and an aggregate commodity index before and after the "financialization" of commodities shows a change from a symmetric dependence structure with increased tail dependence to a constant structure and (iii) the estimation of the dependence of industrial and emerging country equity indices and a global equity index during the global financial crisis in 2007 and 2008 relative to a "normal" period reveals an asymmetric dependence structure in normal times (lower tail dependence) and different structures across industrial and emerging markets in crisis times. More specifically, large industrial markets do not exhibit a change in the degree or structure of dependence whereas emerging markets display a shift from an asymmetric structure to a more constant dependence structure. The differences among markets suggest that contagion is more prevalent in emerging markets while industrial countries only show interdependence.

The analysis of three different asset classes illustrates that the methodology proposed in this paper can detect very different degrees and structures of dependence and also model dynamics in the dependencies, that is identify changes through time.

The remainder of the paper is structured as follows: Section 2 introduces the econometric framework, Section 3 presents the empirical applications and Section 4 summarizes the results and concludes.

2. Econometric framework

The basis of our novel approach is the copula concept which was introduced by Sklar (1959). Suppose that H is a k-dimensional distribution function with one-dimensional marginal distributions $F_1, \ldots, F_k$, then there is a copula C such that

$$H(x_1, \ldots, x_k) = C(F_1(x_1), \ldots, F_k(x_k)). \tag{1}$$

for $x_1, \ldots, x_k \in \overline{R}$

If the marginals are continuous, then C in Eq. (1) is unique and is given by

$$C(u_1, \ldots, u_k) = H(F_1^{-1}(u_1), \ldots, F_k^{-1}(u_k)). \tag{2}$$

where $u_i = F_i(x_i), i = 1, \ldots, k$.

Eqs. (1) and (2) show that a copula is a mapping from $[0, 1]^k$ to $[0, 1]$. In other words, it connects k marginal distributions to form a joint distribution.[2] Since the focus of this study is on the degree and the structure of dependence, we describe these components implicit in a multivariate distribution with the example of a Gaussian copula. The bivariate Gaussian copula C_G is defined by

$$C_G(u, v, \rho) = \mathbf{\Phi}_\rho(\Phi^{-1}(u), \Phi^{-1}(v)) \tag{3}$$

where Φ is the univariate normal distribution, and $\mathbf{\Phi}_\rho$ is the standardized bivariate normal distribution with correlation ρ.

As pointed out and demonstrated by Hu (2006), the copula facilitates the separation of the degree and the structure of dependence. For a multivariate Gaussian distribution $\mathbf{\Phi}_\rho$, the degree of dependence is given by ρ and the structure is represented by the Gaussian distribution or the copula C_G. Hu (2006) proposes a mixture model to better account for the typical dependence characteristics of international stock markets. She uses a Gaussian copula, a Gumbel copula and a Gumbel survival copula. The Gaussian dependence structure is symmetric while the Gumbel copula is asymmetric and allows to generate right-tail dependence. The Gumbel survival copula is also asymmetric and models left-tail dependence. The mixture of the Gaussian, the Gumbel and the survival Gumbel with weights w_1, w_2 and $1 - (w_1 + w_2)$ is given by

$$C_{mix}(u, v; \rho, \alpha, \beta, w_1, w_2) = w_1 C_G(u, v, \rho) + w_2 C_{GU}(u, v; \alpha) + (1 - w_1 - w_2) C_{GU^*}(u, v; \beta) \tag{4}$$

Here, α and β represent the dependence parameter in the Gumbel and survival Gumbel copula, respectively. The dependence parameters govern the degree of dependence for a given structure implied by the copula.

Rodriguez (2007) also employs a mixture of copulas but uses the Frank, Clayton and Gumbel copula where the weight of each copula in the mixture depends on specific states according to a Markov Switching process. For example, if the weights of the Clayton and the Gumbel copula increase from a low variance state to a high variance state, both lower and upper tail dependence increase in high variance states.[3]

Hu (2006) and Rodriguez (2007) among many others specify the mixture of copulas to obtain a joint distribution that provides the best fit to the data. Despite the fact that the copula joins the marginal distributions to build a joint distribution the focus of most papers in the empirical finance literature is on the copula and not on the joint distribution. Given this, we propose an alternative to the copula framework which is based on a quantile regression model as introduced by Koenker and Bassett (1978). The model is specified as follows

$$y = x'\beta + \epsilon \quad \text{with} \quad Q_y(\tau|x) = x'\beta(\tau) \tag{5}$$

where $Q_y(\tau|x)$ denotes the τth conditional quantile of y, assumed to be linearly dependent on x. The dependence is given by $\beta(\tau)$ and can be interpreted as 'unconditional' if no exogenous (control) variables are included in the model given by Eq. (5). Conversely, the dependence can be viewed as 'conditional' if exogenous (control) variables are added to the model.[4] The quantile-based dependence vector $\boldsymbol{\beta}(\tau)$ connects y and x similar to the copula $\mathbf{\Phi}_\rho$ which connects $\Phi^{-1}(u)$ and $\Phi^{-1}(v)$ as in Eq. (3). The difference is that the quantile

[2] More details are provided in Nelsen (1999) for example.

[3] See Embrechts et al. (2002, 2003) for applications of tail dependence in risk management.

[4] Copulas are often fitted based on filtered data (in a first stage) and thus display the conditional dependence of the variables (e.g. see the empirical illustration in Patton, 2012).