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Inference in asset pricing models with a low-variance factor

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ABSTRACT

This paper concerns with the effects of including a low-variance factor in an asset pricing model. When a low-variance factor is present, the commonly applied Fama–MacBeth two-pass regression procedure is very likely to yield misleading results. Local asymptotic analysis and simulation evidence indicate that the risk premiums corresponding to all factors are very likely to be unreliably estimated. Moreover, *t*- and *F*-statistics are less likely to detect whether the risk premiums are significantly different from zero. We recommend Kleibergen's (2009) *FAR* statistic when there is a low-variance factor included in an asset pricing model.

1. Introduction

Asset pricing and portfolio management are the main building blocks of modern finance theory. Understanding how to price an asset and how to construct optimal portfolios has important practical implications for both policy makers and practitioners. Asset pricing theories typically model expected returns as a linear function of systematic risks and risk premiums corresponding to some macroeconomic or financial factors. An interesting aspect that emerges from the descriptive analysis of factor models is that the variability of some macroeconomic factors (e.g., the default premium, consumption and labor growth rate, etc.) is very small compared to the variability of asset returns. We refer to the small relative variability of the factors as low signal-to-noise ratio (SNR). In Fig. 1, we plot the returns of two portfolios selected from the Fama–French 10 portfolios formed on size along with the growth rate of per capita US labor income from July 1963 to December 2011.[1] The return series in the top and bottom figures are from the portfolios with the smallest and largest variances among the 10 portfolios, respectively. The smallest and largest variance portfolios are corresponding to the 10th decile and 1st decile portfolios, respectively. Evidence from Fig. 1 shows that even the returns with the smallest variance are much more volatile than the labor factor.

The commonly applied methodology for estimating the models is the Fama and MacBeth (FM) procedure, proposed by Fama and MacBeth (1973). In the first pass, regress asset returns on the factors to obtain the coefficients, betas. In the second pass, the estimated betas become the regressors, and we calculate the corresponding coefficients, the risk premiums.

Given a correctly specified model, the large sample inference on the estimated risk premiums can approximate their finite sample behavior well. Over the last 20 years, researchers in financial economics have been actively exploring properties of the FM procedure on estimation and testing asset pricing models under model misspecification, the error-in-variable (EIV) problem, and possibly irrelevant factors. For example, the FM procedure treats the estimated betas as the true betas in the second-pass regression, which causes an EIV problem. Shanken (1992) analyzes the asymptotic properties of the estimated risk premiums by taking account of the EIV problem under conditional homoskedastic error terms. He argues that the usual FM standard errors are incorrect and proposes the EIV-adjusted standard errors. Later, Jagannathan and Wang (1998) extend the properties to the case of weakly stationary and ergodic errors. From a model misspecification perspective, Kan and Zhang (1999) investigate the properties of the two-pass procedure when a factor is independent of the asset returns. They argue that the *t*-statistic tends to over-reject the null and lead to the conclusion that the factor is useful. Kleibergen (2009) generalizes Kan and Zhang (1999) and argues that the *t*-statistic is unreliable when the true betas associated with the factors are either zero or close to zero. He also proposes some new test statistics, including the factor Anderson–Rubin statistic (*FAR*).

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[1] The 10 size portfolios are constructed from all NYSE, AMEX, and NASDAQ stocks using the June market equity and NYSE breakpoints. They appear in Kenneth French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french.

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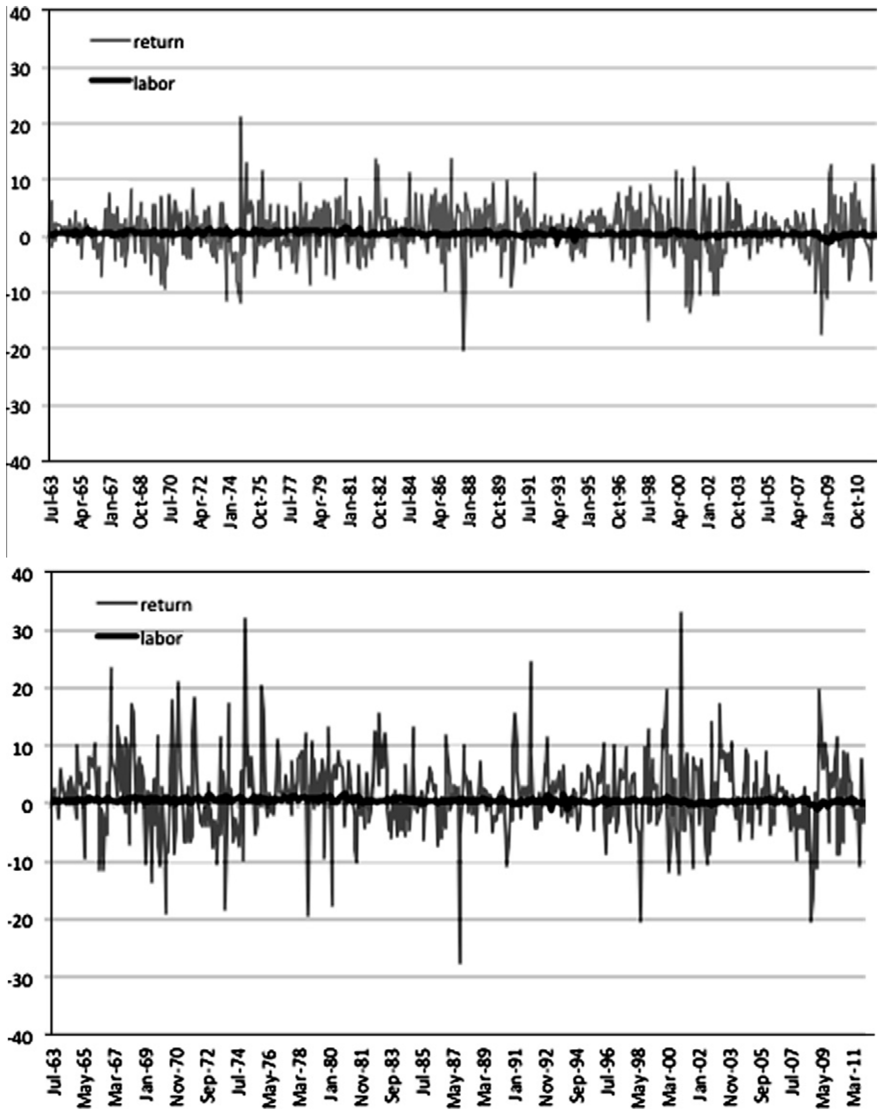

Fig. 1. The relative variation of the returns of Fama and French 10 portfolios formed on size and the growth rate of per capita US labor income from July 1963 to December 2011. (a) Relative variation of returns with the smallest variance and labor, (b) Relative variation of returns with the largest variance and labor.

There is little discussion about the effect on estimation and test statistics under a low SNR. In a framework of predicting asset returns using an explanatory variable, Torous and Valkanov (2000) argue that a low SNR renders unreliable estimation, inference, and forecasting. Gospodinov (2009) shows that the low relative variation of the forward premium and the exchange rate returns creates a large bias and variability of the estimated slope parameters in a differenced forward premium regression, and leads to size distortions of the t-statistic. Chen and Kan (2005) point out that in a one-factor linear asset pricing model, the magnitude of the finite sample percentage biases of the estimated zero-beta rate and the risk premium by GLS, is an inverse function of the relative variance of the true betas and the estimated betas.

To the best of our knowledge, this is the first paper that investigates the properties of the two-pass regression when a low-variance factor, which leads to a small SNR, is present in an asset pricing model. We analyze the properties of the estimated parameters corresponding to both the low-variance factor and the other factors. Our first contribution is that we use local-asymptotic analysis to approximate the finite sample behavior of the two-pass estimator when a model includes a low-variance factor. The local-asymptotic analysis yields a better approximation for the finite sample properties of a test. In order to account for the low variability of the factor, we parameterize its variance as the product of a constant (called a localizing constant) and the error variance dividing the sample size. This provides us with a convenient tool to derive the distribution of the estimates when the variance of the factor drifts to zero. It also implies that the information used to estimate the parameters provided by the low-variance factor remains low when the sample size increases. The standard asymptotics assume that the information increases as the sample size increases. Thus, the estimates become less volatile with time. In order to control other determinants, for example, heteroskedasticity and autocorrelation, which possibly influence the estimator, and concentrate on the effect of the variance, we derive the distribution of the Fama–MacBeth two-pass estimator assuming the error term is independent and identically distributed (i.i.d.) across time and assets. The distribution under heteroskedasticity and autocorrelation is derivable in a similar fashion as in Shanken (1992) or Jagannathan and Wang (1998). We expect that the results are similar to those under the stronger assumptions.

Afterwards, we analyze the actual finite sample properties of this estimator via simulation. We also compare the performances of the t-statistics, F-statistics, and Kleibergen's FAR statistic. Our