Journal of Banking & Finance 34 (2010) 2742–2752

Contents lists available at ScienceDirect

Journal of Banking & Finance

journal homepage: www.elsevier.com/locate/jbf

Commodity derivatives valuation with autoregressive and moving average components in the price dynamics

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ARTICLE INFO

Article history:
Received 15 December 2009
Accepted 16 May 2010
Available online 21 May 2010

JEL classification:
G13
C50
Q40

Keywords:
Commodity pricing
CARMA
Futures
Crude oil

ABSTRACT

In this paper, we develop a continuous time factor model of commodity prices that allows for higher-order autoregressive and moving average components. We document the need for these components by analyzing the convenience yield's time series dynamics. The model we propose is analytically tractable and allows us to derive closed-form pricing formulas for futures and options. Empirically, we estimate a parsimonious version of the general model for the crude oil futures market and demonstrate the model's superior performance in pricing nearby futures contracts in- and out-of-sample. Most notably, the model substantially improves the pricing of long-horizon contracts with information from the short end of the futures curve.

1. Introduction

Commodity prices and their stochastic behavior play a central role in many economic and financial decisions. The valuation of commodity contingent claims is different than the valuation of pure financial contracts as the underlying assets are held for consumption and production. The theory of storage implies that the owner of the commodity receives a benefit from holding a stock of commodities since this enables him to employ them the moment they are needed, thus saving cost, trouble, and time involved with ordering new supplies. This real option, usually referred to as *convenience yield*, complicates the valuation of even the simplest derivatives contracts, as its value is not readily observable.[1] Thus, simple cost-of-carry arguments, as used for financial contracts, are not easily applicable.

Brennan and Schwartz (1985) is one of the first works proposing to employ financial modeling techniques for commodity prices to value commodity contingent claims via arbitrage methods. They model the convenience yield as a pure function of the spot commodity price. However, this rather strong assumption yields inferior empirical results; see, for example, Brennan (1991). Gibson and Schwartz (1990) therefore extend the Brennan-Schwartz approach by modeling the convenience yield itself as a second stochastic factor, improving the model's empirical performance. Schwartz and Smith (2000) reformulate the Gibson–Schwartz model as an equivalent latent factor model that is easier to work with from an econometric point of view. Other studies extend these models to include even more stochastic factors; for example Schwartz (1997), Casassus and Collin-Dufresne (2005), Geman and Nguyen (2005). It remains controversial, however, whether a third factor can improve the models' performance or merely yields overparameterization.

In this paper, we take a different, more parsimonious approach than simply adding additional stochastic factors. Extant studies assume (explicitly or implicitly) that the convenience yield follows an Ornstein–Uhlenbeck type process, which is the continuous limit of a discrete *AR(1)* process. Nevertheless, when analyzing the convenience yield, we find in a preliminary analysis of crude oil futures data that this assumption is not very satisfactory from an empirical point of view (see Section 2). Adding a moving average component yielding an *ARMA(1,1)* model, however, improves the statistical description of the convenience yield's dynamics significantly. Consequently, we propose to include this empirical feature in a continuous time commodity pricing model.

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[1] See Kaldor (1939), Working (1949), or Brennan (1958), for a detailed discussion of the theory of storage and the arising convenience yield.

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doi:10.1016/j.jbankfin.2010.05.010

Our main contribution is twofold. First, theoretically, we develop a continuous time commodity pricing model that is able to incorporate higher-order autoregressive and moving average terms. This enables us to capture the stylized facts observed for the convenience yield without the need to add additional risk factors or to leave the Gaussian framework. The latter fact allows us to derive closed-form futures and options valuation formulas. Second, empirically, we implement a parsimonious specification of our model for the crude oil futures market. A comparison with the benchmark model of Schwartz and Smith (2000), shows that the proposed model greatly improves the futures pricing at the short end of the futures curve both in- and out-of-sample. Most notably, the model also substantially improves the pricing of long-horizon contracts with information from the short end of the futures curve.

Our model can be regarded as a generalization of the well-known model of Schwartz and Smith (2000). We follow their approach and do not consider an explicit convenience yield, but rather formulate the model in a latent factor form which facilitates empirical implementation. Schwartz and Smith (2000) assume in their model that the second factor, describing short-term deviations from the long-term equilibrium price, follows an Ornstein–Uhlenbeck process. We generalize this approach by replacing it with a continuous autoregressive moving average (*CARMA*) process. *CARMA* processes have been studied in the statistical literature for a long time (see Tsai and Chan, 2000 or Brockwell, 2001 and the references therein) but have received very little attention in financial modeling. Only Benth et al. (2008) have recently proposed using *CARMA* processes for interest rate modeling and discuss the merits of this approach.

The properties of the *CARMA* process, compared to the simple Ornstein–Uhlenbeck process, are very desirable to model commodity futures prices.[2] First, adding higher-order autoregressive and, more importantly, moving average components, to the model allows much more flexibility with respect to the shape of the futures curve; and, second, the term structure of volatilities. As a consequence, it is able to yield a much better pricing performance. This is especially true for the short end of the futures curve, usually the worst part of the curve with respect to pricing accuracy, because of the very high volatility of the nearby contracts.

The remainder of this paper is structured as follows. In Section 2, we conduct a preliminary analysis of crude oil futures prices to motivate our model. In Section 3, we introduce the *CARMA* process in general, derive our commodity pricing model, and discuss its properties. In Section 4, we describe the Kalman filter-based estimation of the model, whereas Section 5 presents our empirical study of crude oil futures. Concluding remarks are provided in Section 6.

2. Preliminary data analysis

2.1. Data

Our data set consists of prices of crude oil futures contracts traded on the New York Mercantile Exchange (NYMEX), one of the most heavily traded commodity contracts worldwide.[3] The short position in this contract commits the holder to deliver 1000 barrels of domestic crude oil in Cushing, Oklahoma.[4] We consider weekly observations, sampling Wednesday settlement prices between 01/01/1996 and 12/10/2008, yielding 676 observation dates. Crude oil futures are listed nine years forward with monthly maturity for the first six years and semiannual maturity thereafter. As liquidity is rather low for longer-term contracts, we consider only the first 24 contracts (that is, the first two years) in our analysis. Thus we employ a total of 16,224 futures prices. We conduct our study using settlement values of futures prices, as they are classically considered to be representative for a trading day (see Geman and Nguyen, 2005; Marshall et al., 2008). As maturity, we use the last day of trading.[5] We obtain all data from Bloomberg.

Table 1
Statistics of crude oil futures contracts: This table reports statistics for weekly observations of crude oil futures contracts from 3 January, 1996 to 10 December, 2008. Prices are in dollars per barrel. F01 denotes the one-month futures contract, F02 the two-month contract and so on. SD denotes the standard deviation.

	Mean price	SD	Maturity	SD
F01	40.43	26.48	0.0450	0.0243
F02	40.42	26.65	0.1284	0.0243
F03	40.35	26.79	0.2119	0.0244
F04	40.24	26.91	0.2951	0.0244
F05	40.11	27.01	0.3785	0.0243
F06	39.98	27.09	0.4620	0.0244
F07	39.84	27.16	0.5453	0.0244
F08	39.71	27.22	0.6288	0.0244
F09	39.58	27.26	0.7121	0.0245
F10	39.45	27.29	0.7954	0.0244
F11	39.33	27.31	0.8789	0.0243
F12	39.22	27.33	0.9623	0.0244
F13	39.10	27.34	1.0456	0.0244
F14	38.99	27.34	1.1291	0.0244
F15	38.89	27.35	1.2124	0.0244
F16	38.79	27.35	1.2958	0.0243
F17	38.70	27.34	1.3793	0.0244
F18	38.61	27.33	1.4626	0.0245
F19	38.53	27.32	1.5460	0.0244
F20	38.45	27.31	1.6295	0.0244
F21	38.38	27.30	1.7128	0.0245
F22	38.31	27.28	1.7961	0.0244
F23	38.25	27.26	1.8796	0.0244
F24	38.19	27.25	1.9628	0.0243

Table 1 contains summary statistics for the futures price data, where F01 is the contract closest to maturity, F02 the second contract closest to maturity and so on. In line with prior research, the average futures curve is in backwardation, though at a much higher level, which is mainly because of the peak of the most recent observations. Fig. 1 provides a time series plot of the closest-to-maturity futures contract F01.

2.2. Convenience yield

In this subsection, we conduct a preliminary analysis of the convenience yield in the crude oil market to motivate our new model. This analysis is complicated by the fact that the convenience yield is not observable. Thus we have to rely on some approximation. Using the well-known relationship between spot and futures prices when storage costs s_t, interest rates r_t, and net convenience yields c_t are constant

$$F(t,T) = S_t e^{(s_t + r_t - c_t)(T-t)} = S_t e^{\delta_t (T-t)}, \tag{1}$$

enables us to estimate monthly forward total convenience yields δ_t. The total convenience yield already includes the costs of storage and capital. As we do not have spot price data corresponding to the futures data, we follow the approach taken by Gibson and Schwartz (1990), Lien and Yang (2008) and use the two futures con-

[2] Note that the Ornstein–Uhlenbeck process is a special case of a *CARMA* process.

[3] The NYMEX crude oil futures contract is used in many other studies, e.g., Schwartz (1997), Schwartz and Smith (2000), Cortazar and Naranjo (2006), Geman and Kharoubi (2008), Doran and Ronn (2008).

[4] The following domestic oil grades are deliverable: West Texas Intermediate, Low Sweet Mix, New Mexican Sweet, North Texas Sweet, Oklahoma Sweet and South Texas Sweet. Specific foreign crudes may also be deliverable, however, at a discount. For details on the specification of deliverable crudes and delivery locations, see www.nymex.com.

[5] Trading ends at the close of business on the third business day prior to the 25th calendar day of the month preceding the delivery month. If the 25th calendar day of the month is a non-business day, trading shall cease on the third business day prior to the business day preceding the 25th calendar day.