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Informed arbitrage with speculative noise trading

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ABSTRACT

We consider speculative noise trading when some naïve speculators trade on noise as if it were information [Black, F., 1986. Noise. Journal of Finance 41, 529–543]. We examine the optimal trading strategy of an informed investor who faces such naïve speculators in the market. We find that the informed investor trades aggressively on her information and takes large, opposite positions against the naïve speculators. The trading volume is thereby drastically magnified. While such speculative noise trading enhances liquidity, it makes prices less efficient. The overall dynamic patterns that emerge from our model are most consistent with the evidence for interday variations in volume, volatility, and transaction costs.

1. Introduction

In a seminal paper, Kyle (1985) analyzes how an insider carefully exploits his informational advantage over other market participants through trading. In that setting, while other traders do not have access to the private information, they nonetheless agree with the insider about the asset's underlying valuation model. In practice, traders may employ different valuation models as they focus on distinct pieces of information. In particular, some traders may focus on non-fundamental information such as technical analysis or rumors heard on the street. Black (1986) argues that this kind of noise trading – trading on noise (or rumor) as if it were information – is pivotal to explaining the phenomenon of high volume observed in financial markets. This kind of speculative noise trading by "naïve" speculators is absent from the original model of Kyle (1985).

In this paper, we undertake a formal analysis on the effects of such speculative noise trading in the financial markets. To do so, we extend the model of Kyle (1985) by allowing for two alternative valuation models, U and V, regarding the liquidation value of the asset, $\tilde{d}$. The U model is identified with a rumor-based signal $\tilde{u}$ and the V model is identified with a fundamental-based signal $\tilde{v}$ for the liquidation value, $\tilde{d}$. At the end of the trading period, the liquidation value is revealed and set by the V model, i.e., $\tilde{d} = \tilde{v}$. In this scenario, we examine how an informed investor who observes both signals $\tilde{u}$ and $\tilde{v}$ should optimally trade when facing a group of naïve speculators who trade on signal $\tilde{u}$.

In this richer setting, we find that the optimal strategy of the informed trader differs significantly from Kyle (1985). For example, it is well known that the insider in Kyle (1985) refrains from aggressive trading and camouflages his information within the uninformed liquidity trading. By contrast, the informed trader in our model not only trades aggressively on her information, but also takes large, opposite positions against naïve speculators operating under heterogeneous beliefs. Consequently, trading volume is significantly magnified – a result that confirms the conjecture in Black (1986) on the role of speculative noise trading in explaining the high volume phenomenon.

Our result shows that the informed trader uses both signals, $\tilde{u}$ and $\tilde{v}$, in trading, despite the fact that liquidation value is determined solely by the fundamental-based signal, $\tilde{v}$. However, the informed investor chooses not to completely eliminate the influence of speculative noise trading on prices. Through the act of trading itself, the informed trader imparts information into prices, while naïve speculators impart noise. In a dynamic model, the

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informativeness of prices becomes less than that in Kyle (1985), as the amount of noise from speculative noise trading eventually outweighs the information imparted from informed trading. This result confirms Black's (1986) conjecture that speculative noise trading leads to a more liquid market with higher trading volume, but it also makes prices less efficient.

Finally, our model yields useful implications for trading dynamics over time. Among them, the overall dynamic patterns that emerge from our model are most consistent with the empirical evidence for interday variations in volume, volatility, and transaction costs (Jain and Joh, 1988; Foster and Viswanathan, 1993).

This paper proceeds as follows: In Section 2, we discuss a one-shot trading model and its properties in order to motivate a dynamic model that follows. Section 3 describes the dynamic model and the necessary and sufficient conditions for equilibrium. Section 4 discusses the properties and implications of the dynamic model, based on numerical results. Section 5 presents our conclusions. Proofs are set forth in the Appendix.

2. The one-shot model

The basic setup here is similar to the one-shot model of Kyle (1985), except that we allow for two alternative valuation models, U and V, where the U model is identified with a rumor-based signal $\tilde{u}$ and the V model is identified with a fundamental-based signal $\tilde{v}$ for the liquidation value, $\tilde{d}$. At the end of the trading period, the liquidation value is revealed and set by the V model, i.e., $\tilde{d} = \tilde{v}$.[1] There are four kinds of risk-neutral market participants: (1) an informed trader ("v-trader") who observes both signals and correctly believes that the liquidation value is to be set by the V model, i.e., $\tilde{d} = \tilde{v}$, (2) a naïve trader ("u-trader") who focuses solely on signal $\tilde{u}$, (3) many competitive market makers who observe neither $\tilde{u}$ nor $\tilde{v}$, and (4) many liquidity traders.[2] To emphasize that the two signals $\tilde{u}$ and $\tilde{v}$ reflect two distinct sources of information, assume that $\tilde{u}$ and $\tilde{v}$ are independently and normally distributed with zero mean and identical variances, denoted by $\Lambda_0 \equiv Var(\tilde{u}) = Var(\tilde{v}) \equiv \Sigma_0$ and $\Omega_0 \equiv Cov(\tilde{u}, \tilde{v}) = 0$.

At the beginning of the trading period, the u-trader submits a quantity $\tilde{y}$ given signal $\tilde{u}$ alone, whereas the v-trader submits a quantity $\tilde{x}$ given the two signals $\tilde{u}$ and $\tilde{v}$. Liquidity traders as a group submit an exogenous quantity $\tilde{z}$, which is normally distributed with zero mean and variance σ_z^2. Competitive market makers observe the combined order flow $\tilde{w} = \tilde{x} + \tilde{y} + \tilde{z}$, and set price $\tilde{p}$ semi-strong efficiently to clear the market such that

$$\tilde{p} = E[\tilde{v} | \tilde{w} = \tilde{x} + \tilde{y} + \tilde{z}], \tag{1}$$

where E is the rational expectations operator.

The u-trader knows that he only observes signal $\tilde{u}$ and that the informed trader observes both signals $\tilde{u}$ and $\tilde{v}$. Furthermore, the u-trader mistakenly believes that signal $\tilde{u}$ is the true liquidation value and signal $\tilde{v}$ is pure noise.

Taking into account the pricing rule in (1), the u-trader solves for his optimal strategy, $\tilde{y}$, to maximize his expected profit, given his irrational belief and his observation of signal $\tilde{u}$, i.e.,

$$\underset{y}{Max}\ E^u[(\tilde{u} - \tilde{p})y | \tilde{u} = u]. \tag{2}$$

Note that we distinguish the u-trader's irrational expectations operator, denoted by E^u, from the rational expectations operator, E, used by both the informed trader and the market makers.

By contrast, the v-trader solves for her optimal strategy, $\tilde{x}$, to maximize her expected profit, given her rational belief and her observation of both signals $\tilde{u}$ and $\tilde{v}$, i.e.,

$$\underset{x}{Max}\ E[(\tilde{v} - \tilde{p})x | \tilde{u} = u, \tilde{v} = v]. \tag{3}$$

Following Kyle and Wang (1997), given the agreement to disagree between the two traders, we consider a Bayesian Nash equilibrium in which each trader's conjectured strategy with regard to the other trader is consistent with the strategy that emerges in equilibrium. This one-shot model has a unique equilibrium, as shown in the following proposition.

Proposition 1. *There exists a unique Bayesian Nash equilibrium in which $\tilde{x}$, $\tilde{y}$, and $\tilde{p}$ are linear functions in their respective signals as follows*:

$$\tilde{x} = \beta\tilde{v} + \theta\tilde{u}, \tag{4}$$

$$\tilde{y} = \gamma \cdot \tilde{u}, \tag{5}$$

$$\tilde{p} = \lambda \cdot \tilde{w}, \tag{6}$$

where the constants β, θ, γ, and λ are given by

$$\beta = \frac{1}{2\lambda}, \tag{7}$$

$$\theta = -\frac{1}{3\lambda}, \tag{8}$$

$$\gamma = \frac{2}{3\lambda}, \tag{9}$$

$$\lambda = \frac{\sqrt{5}}{6}(\Sigma_0/\sigma_z^2)^{1/2}. \tag{10}$$

The equilibrium indicates that although the v-trader believes that the liquidation value is set only by the V model, i.e., $\tilde{d} = \tilde{v}$, she will not simply discard the rumor-based signal, $\tilde{u}$, and trade on the fundamental-based signal, $\tilde{v}$, alone. Instead, the v-trader takes a short position to partially offset the u-trader's long position in the signal $\tilde{u}$ (see Eqs. (8) and (9)). Consequently, the price will incorporate influences from both signals $\tilde{u}$ and $\tilde{v}$, as well as the liquidity trade, $\tilde{z}$, i.e.,

$$\tilde{p} = \frac{1}{2}\tilde{v} + \frac{1}{3}\tilde{u} + \lambda\tilde{z}. \tag{11}$$

By contrast, without signal $\tilde{u}$, the asset price and the liquidity parameter under the one-shot model of Kyle (1985), denoted by $\tilde{p}^*$ and λ^*, respectively, are given by

$$\tilde{p}^* = \frac{1}{2}\tilde{v} + \lambda^*\tilde{z}, \quad \text{where } \lambda^* = \frac{1}{2}(\Sigma_0/\sigma_z^2)^{1/2}. \tag{12}$$

In one sense, the v-trader's choice of not fully eliminating the influence of the rumor-based signal, $\tilde{u}$, is tantamount to throwing sand into the eyes of uninformed market makers who are trying to infer information, $\tilde{v}$, from the combined order flow, $\tilde{w}$. The market makers therefore face an additional source of noise from the influence of signal $\tilde{u}$. Nonetheless, Eqs. (11) and (12) show that exactly one-half of the true private information, i.e., $\frac{1}{2}\tilde{v}$, is incorporated into prices in both models. This result strengthens the finding in Kyle (1985) that it is optimal (i.e., profit maximization) for a single insider to reveal exactly one-half of her private information to the market in a one-shot trading, regardless of the amount of noise in the market.

Given the equilibrium in Proposition 1, we compare our one-shot model to that of Kyle (1985) by examining a number of comparative statics as shown in Corollary 1.

[1] Following Kyle (1985), we assume perfect information to simplify our analysis, and the results are qualitatively the same if we assume imperfect information, i.e., $\tilde{v} = \tilde{d} + \tilde{e}$, where $\tilde{e}$ is a white noise.

[2] There is by now a large body of behavioral finance literature documenting the behavior of irrational naïve traders under heterogeneous prior beliefs or quasi-rational agents under bounded rationality, and the naïve u-trader considered in this paper is in line with this literature (see, Kahneman et al., 1982; Shiller, 1989, 2000; Thaler, 1993; Shefrin, 2000; Shleifer, 2000).