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How accurate is the square-root-of-time rule in scaling tail risk: A global study

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ABSTRACT

The square-root-of-time rule (SRTR) is popular in assessing multi-period VaR; however, it makes several unrealistic assumptions. We examine and reconcile different stylized factors in returns that contribute to the SRTR scaling distortions. In complementing the use of the variance ratio test, we propose a new intuitive subsampling-based test for the overall validity of the SRTR. The results indicate that serial dependence and heavy-tailedness may severely bias the applicability of SRTR, while jumps or volatility clustering may be less relevant. To mitigate the first-order effect from time dependence, we suggest a simple modified-SRTR for scaling tail risks. By examining 47 markets globally, we find the SRTR to be lenient, in that it generally yields downward-biased 10-day and 30-day VaRs, particularly in Eastern Europe, Central-South America, and the Asia Pacific. Nevertheless, accommodating the dependence correction is a notable improvement over the traditional SRTR.

1. Introduction

Following several serious financial crises in little more than a decade, including the Asian Financial Crisis of 1997, the Dot-Com Bubble of 2000, and the Global Financial Tsunami of 2008, risk management, particularly in relation to tail risks, has recently increased considerably in importance in numerous subfields of finance. Value at Risk (VaR), defined as a worst case scenario in terms of losses on a typical day, is a popular measure of tail risk management that is not only recommended by banking supervisors (BCBS, 1996a), but is also widely used throughout the financial industry, including by banks and investment funds, see Pérignon and Smith (2010a,b). It is even used by nonfinancial corporations in supervising in-house financial risks following the success of the J.P. Morgan RiskMetrics system.

Operationally, tail risk such as VaR is generally assessed using a 1-day horizon, and short-horizon risk measures are converted to longer horizons. A common rule of thumb, borrowed from the time scaling of volatility, is the square-root-of-time rule (hereafter the SRTR), according to which the time-aggregated financial risk is scaled by the square root of the length of the time interval, just as in the Black–Scholes formula where the T-period volatility is given by $\sigma\sqrt{T}$. Regulators also advocate the routine use of the SRTR. For example, to avoid duplication of risk measurement systems, financial institutions are allowed to derive their two-week VaR measure by scaling up the daily VaR by SRTR; see, for example, BCBS (1996b). In fact, horizons of up to a year are not uncommon; many banks link trading volatility measurement to internal capital allocation and risk-adjusted performance measurement schemes, which rely on annual volatility estimates by scaling 1-day volatility by $\sqrt{252}$.

If the SRTR is to serve as a good approximation of all quantiles and horizons, it not only requires the iid property of zero-mean returns, but also that of the Normality of the returns. These preassumptions are far from being realized in real world financial asset returns, provided the numerous documented stylized facts that are conflict with these properties. Accordingly, numerous studies have attempted to identify how these different effects give rise to bias in SRTR approximation. The first attempt is based on the

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fact that asset returns may be weakly dependent, both in levels and higher moments. As illustrated in Jorion (2001), the SRTR tends to understate long-term tail risk when the return follows a persistent pattern, but tends to overstate the tail risk of temporally-aggregated returns if it displays mean-reverting behavior. Similarly, the presence of *volatility clustering*, as well-documented in the case of most financial assets since Engle (1982), Bollerslev et al. (1992), Bollerslev et al. (1994), under the dynamic setup, has been demonstrated using detailed examples of how the common practice of converting 1-day volatility estimates to h-day estimates by SRTR scaling is inappropriate and yields overestimates of the variability of long-horizon volatility. On this, see Diebold et al. (1997) and Müller et al. (1990).

Numerous extant studies have demonstrated that asset returns exhibit heavy-tails (Fama, 1965; Jansen and de Vries, 1991; Pagan, 1996). Although allowing for dynamic dependence in the conditional variance partially contributes to the leptokurtic nature, the GARCH effect alone does not explain the excess kurtosis in financial asset returns. On the one hand, this motivates studies to employ their empirical GARCH modeling with student-t or generalized error distributions to account for heavier tails. On the other hand, researchers have turned to models that generate price discontinuities to resolve the empirical regularity. Researchers have long realized that financial time series exhibit certain unusual and extreme violent movements, known as *jumps* and modeled using jump diffusions developed by Merton (1976) that create discontinuous sample paths. See Andersen et al. (2002), Pan (2002), Eraker et al. (2003), Becker et al. (2009), Câmara (2009) for recent evidence on the prevailing phenomena of jumps in price processes. Nonetheless, how the underlying jumps influence the SRTR approximation of longer-term tail risks remained unclear until the work of Danielsson and Zigrand (2006). They intuitively and clearly show that SRTR tends to underestimate the time-aggregated VaR and the downward bias deteriorates with the time horizon owing to the existence of negative jumps. However, it remains unseen if in general price jumps are not confined to downside extreme losses only, would the SRTR-induced downward-bias move in the other direction instead or become negligible?

Although we sound different alarms from distinct perspectives by disclosing SRTR scaling as being inappropriate and misleading, with documented upward biases for some effects and downward biases for others, it is unclear after all whether the overall validity of the SRTR is appropriate or not for practical risk implementation given that all these effects coexist in a given asset. However, this paper is not merely concerned with individual effects, such as a weak dependence of returns, volatility scaling, price discontinuities or leptokurticity, as is the case for the literature on the time scaling performance of the SRTR. Instead, we are interested in the interactions among these stylized facts on the scaling of tail risks via the application of the SRTR. To our knowledge, no previous investigation has reconciled the quality of approximation in time-aggregated tail risks using the SRTR under various confounding factors.

This study fills this void by first devising a general framework for disentangling and separately estimating the sensitivity toward each systematic risk factor. To examine the overall performance of the SRTR approximation and characterize the potential bias, we define a bias function using a benchmark VaR based on averaging a set of subsampled non-overlapping temporal aggregated VaRs. Based on Monte Carlo experiments, this investigation demonstrates that dependence at the return level is the dominant bias factor. The SRTR leads to a systematic underestimation (overestimation) of risk when the return follows a persistent (mean-reverting) process, and can do so by a substantial margin. Moreover, the magnitude of downward (upward) bias increases with the time horizon. However, volatility clustering tends to drive the time-aggregated VaR to slightly underestimate its true value. Alternatively, the heavy-tailed nature of the underlying return overstates the time-aggregated VaR via the SRTR. Perhaps surprisingly, unlike the solely unilateral downside jumps specified by Danielsson and Zigrand (2006) that indicate a severe underestimation bias, the Monte Carlo allowing for both sided jumps with Poisson arrival performed in this study suggests that there is a slight overestimation when scaling with the SRTR.

In view of these results, proper tests for a preliminary verification of the applicability of the SRTR in practice are required. This study first recommends a new informal but informative subsampling-based test, complementing the variance ratio test developed by Lo and MacKinlay (1988),[1] for empirical studies. Moreover, it also contributes to the literature by suggesting a simple modified-SRTR that is robust to the time dependence-induced biases. By utilizing 47 markets included in the MSCI index, including both developed and emerging markets, this study demonstrates that the SRTR underestimates 10-day and 30-day VaRs by an average of approximately 5.7% and 13%, respectively. We also observe that the severity of downward bias is greater for emerging markets in Eastern Europe, Central and South America, and the Asia Pacific. For some developed markets, even when the model assumptions are violated, the SRTR scaling yields results that are correct on average, as shown in the global investigation. This occurs because the underestimation resulting from the dynamic dependence structure is counterbalanced by the overestimation resulting from the excess kurtosis and jumps. Hence SRTR scaling can be appropriate in some cases. Although its widespread use as a tool for approximate horizon conversion is understandable, caution is, however, necessary. We believe that the use of certain pretests as we proposed beforehand is important and may illuminate the applicability of SRTR in the practical approximation of tail risks. Our newly-proposed modified-SRTR approach is shown to be effective in alleviating the bias attributable to the first-order effect from time dependence and the dependence correction is a notable improvement over the traditional unadjusted raw SRTR.

The remainder of this paper is organized as follows. Section 2 formally defines the time-aggregated VaR and SRTR scaling. Section 3 then performs algebraic analysis, in conjunction with Monte Carlo simulations, to disentangle each isolated different stylized effect on the SRTR. This section also briefly reviews the variance ratio test devised by Lo and MacKinlay (1988). Section 4 introduces the suggested variance ratio test and a newly-developed subsample-based test for pretesting the applicability of the SRTR. More importantly, we introduce a new tail risk scaling rule–the Modified-SRTR. Section 5 subsequently summarizes the global empirical study based on data from 47 developed and emerging markets included in the MSCI index. Finally, Section 6 presents the conclusions.

2. Time-aggregated value at risk

The 1-day VaR, defined as VaR (1), measures the maximum possible loss over one trading day under a given confidence level $100 \times (1 - c)$. Supposing that the initial investment of the asset is$1 and R is the random rate of return, then, the asset value at the end of this trading day is $v = 1 + R$. Then, the one-day VaR, VaR (1), under $100 \times (1 - c)$ confidence level is defined as

$$\mathrm{VaR}(1) = -\inf\{r|P[R \leqslant r] > c\}. \tag{1}$$

[1] Finding that using SRTR to estimate Sharpe Ratios causes bias when returns exhibit serial dependence, Lo (2002) suggests using the variance ratio test as a pretest. Other related works include Huang (1985) and Ayadi and Pyun (1994), among many others.