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Extending the Basel II approach to estimate capital requirements for equity investments

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ABSTRACT

Under the Basel II banking regulatory capital regime the capital requirements for credit exposures are calculated using the Asymptotic Single Risk Factor (ASRF) approach. The capital requirement is taken to be the contribution of an exposure to the unexpected loss on the bank's diversified portfolio. Here we extend this approach to calculate capital requirements for equity investments. We show that in the case when asset values have a normal distribution an analytical formula for the unexpected loss contribution may be developed. We show that the capital requirements for equity investments are quite different to those of credit exposures, since equity investments can suffer substantial loss of value even when the underlying company has not defaulted.

Unexpected loss is commonly used as a measure of capital requirements, but it ignores the ability of earnings to absorb loss. We propose a definition of capital requirement that recognises the expected earnings on assets, and show how to combine the ASRF model and the Capital Asset Pricing Model to compute this quantity for credit and equity exposures.

1. Introduction

The Basel II capital adequacy framework (Basel Committee on Banking Supervision, 2006) has recently been adopted by many prudential regulators around the world as the basis of their local regulatory capital regimes for banks. In this framework, under the internal ratings-based approaches, the capital requirement for a credit exposure is taken to be the contribution of the exposure to the unexpected loss on the bank's overall portfolio, computed using the asymptotic single risk factor (ASRF) model (Basel Committee on Banking Supervision, 2005). The ASRF model (Vasicek, 1991; Gordy, 2003) is derived by assuming that the bank has a large and granular portfolio of exposures [for technical assumptions see] (Gordy, 2003) and that the assets against which these credit exposures are secured depend on each other only via their correlation with a single systematic risk factor. Under these assumptions one can derive a formula for the unexpected loss on the bank's credit portfolio that can be decomposed as a sum across the individual exposures. This allows one to give a formula for the unexpected loss contribution of each position and then obtain the capital requirement of the bank by adding up these contributions across all the positions. This means that the capital required to support a particular position does not depend on the other positions in the bank's portfolio, as long as the ASRF assumptions are reasonable, which is a great practical advantage.

By unexpected loss at a confidence level we mean the difference between the mean loss and the loss at that confidence level. Under Basel II the unexpected loss contribution for credit exposures is estimated using a 99.9% confidence level and a one-year default probability. For exposures of one year maturity the Basel II formula is that derived from the ASRF model using a single period of length one year. For exposures of other maturities an heuristic maturity adjustment is applied, which increases the capital required for longer-dated exposures.

As well as loans and other credit exposures financial institutions may have equity exposures to a variety of firms. When these are in the trading book the capital to support them is estimated under the market risk framework, typically based on Value-at-Risk. For banking book equity exposures there are a number of approaches set out in Basel II for estimating capital requirements. These are:

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- The simple risk weight approach, under which risk weights of 300% or 400% are applied (hence capital requirements of 24% or 32% of face value[1]).
- The internal model approach, under which loss relative to risk-free returns over 3 months at the 99% confidence level should be estimated using a model constructed by the bank.
- The PD/LGD approach, under which the Basel II credit risk model is applied with a 90% LGD (loss given default), the PD (default probability) of the underlying counterparty (the same PD that would be applied to a loan exposure to that counterparty), and a five-year maturity.

These methods apply subject to certain limits on the size of the portfolio and on individual positions, after which capital deductions must be taken (100% capital requirement).

The parameters of the internal model approach seem to indicate that the Basel committee considers that potential loss over one quarter at the 99% confidence level will yield a level of capital sufficient to support banking-book equity investments. This standard is lower than that applied for credit exposures, where loss over one year at the 99.9% confidence level must be covered.

By contrast the PD/LGD approach seems on the face of it to require loss over one year at 99.9% to be covered for equity exposures also. The requirement to use a 90% LGD is perhaps an acknowledgement that if a company defaults on its debt it is likely that equity holders will lose all or most of their stake. Equity exposures are however fundamentally different to credit exposures in that substantial loss of value may occur even if the underlying firm does not default. Simply applying the Basel II credit risk capital formula will therefore underestimate the capital requirements for equity investments, as it picks up a contribution to unexpected loss only in those states of the world where the counterparty defaults.

In this paper we will assume that the capital for banking-book equity exposures should be calculated to the same standard as is applied to credit exposures. The modelling period and confidence level are typically set by banks as part of their own internal capital adequacy assessment frameworks. To facilitate comparison with the Basel II capital requirements for credit exposures we will take this to be loss over one year at the 99.9% confidence level. We will develop the capital requirement for equity investments under the assumptions of the Basel II credit model, but will compute the loss in a way which reflects the payoff of an equity position, rather than a debt position. As we shall see this is not the same as simply using the Basel II credit risk formulae.

First we will briefly summarise the ASRF approach. Then we will work out the contribution of an equity position to portfolio unexpected loss under this approach. Unexpected loss is commonly used as a measure of capital requirements, but it ignores the ability of earnings to absorb loss. We will propose a definition of capital requirement that recognises the earnings that can be expected on risky assets, and show how to combine the ASRF model and the Capital Asset Pricing Model to compute this quantity for credit and equity exposures.

For further consideration of the Basel II capital framework see for example Ruthenberg and Landskroner (2008); Jokipii and Milne (2008); Blum (2008) and Heid (2007).

2. The ASRF capital model

We consider a situation in which a bank has made loans to or equity investments in a variety of firms. We wish to compute the value of this portfolio at the end of a single period lasting from time 0 to time T. We will denote by $A_{j,T}$ the value of the assets of firm j at time T, and by Y_j the standardised asset value (so $Y_j = (A_{j,T} - \mu_j)/\sigma_j$, where μ_j and σ_j are the mean and standard deviation of A_j). We assume the dependency between assets is driven by a single systematic risk factor, denoted X, as follows:

$$Y_j = \rho_j X + \sqrt{1-\rho_j^2}\epsilon_j, \quad (1)$$

where ϵ_j is a risk factor specific to firm j, assumed to be independent of X and of ϵ_i for each other firm i. The variables X and ϵ_j are assumed to be standardised also (mean zero, standard deviation one), but at this point we make no other assumptions about their distributions.

The correlation between $A_{j,T}$ and the systematic risk factor X is then ρ_j (we will refer to this as the *asset-factor* or *asset-market* correlation), and the correlation between the assets of firm i and firm j is $\rho_i\rho_j$. The factor R in the Basel II credit formulae is ρ_j^2 (Basel Committee on Banking Supervision, 2005) which can be thought of as an *asset-asset* correlation, as if two assets have $\rho_j = \rho_i = \rho$ the correlation between them will be ρ^2.

Structural or Merton-style models of credit risk assume that default occurs when the a firm's asset value at the end of the period drops below some threshold (which can be thought of as the face value of the firm's debt) (see for example Schönbucher, 2003). Vasicek (1991) derives a formula for the portfolio credit loss distribution in a structural credit risk model where the assets of the counterparties have a single-factor correlation structure and the portfolio is large. Gordy (2003) generalises Vasicek's result. He shows that if losses per dollar of exposure are bounded, mutually independent conditional on the single systematic risk factor X, and the exposure sizes satisfy a condition that says the size of the single largest exposure as a percentage of the portfolio vanishes as the portfolio becomes large, then as the portfolio becomes large the distribution of portfolio loss rate degenerates to its expectation conditional upon X.

Write $G_{j,T}$ for the payoff of position j, some function of the underlying standardised asset payoff Y_j.[2] The portfolio payoff is then $P_T = \sum_j G_{j,T}$. Then under Gordy's assumptions the α-percentile of the portfolio payoff, P_T^α, may be computed as a conditional expectation:

$$P_T^\alpha = \mathrm{E}(P_T|X = X^\alpha),$$

where X^α is the α-percentile of the systematic risk factor X. Expressing the percentiles of the portfolio loss distribution as conditional expectations means they may be decomposed as sums over the positions:

$$P_T^\alpha = \sum_j \mathrm{E}(G_{j,T}|X = X^\alpha).$$

The *unexpected loss* on the portfolio at confidence level $1-\alpha$ is then

$$\mathrm{E}(P_T) - P_T^\alpha = \sum_j \mathrm{E}(G_{j,T}) - \mathrm{E}(G_{j,T}|X = X^\alpha). \quad (2)$$

The term $\mathrm{E}(G_{j,T}) - \mathrm{E}(G_{j,T}|X = X^\alpha)$ can then be thought of as the *unexpected loss contribution* of position j. It is the incremental unexpected loss that arises from adding position j to the portfolio, assuming the portfolio is already well-diversified. Under Gordy's assumptions it is computed as the difference between the expected

[1] In the Basel II framework each exposure is multiplied by a risk weight, with minimum required capital being 8% of risk-weighted assets.

[2] Gordy works with loss rates but the notion of loss doesn't arise naturally for equity positions. We will work with the *payoff*, or end-of-period value, and make the convention that greater values of $G_{j,T}$ mean more money returning to the bank. For credit positions the *loss* is the difference between what was promised and the payoff.