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# What motivates a subprime borrower to default?

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ABSTRACT

This paper uses a real options approach to analyse the exercise of the default option embedded in mortgages. In particular, it examines a subprime household who borrows at a premium, but hopes to refinance at prime rates if their house appreciates. We show how these optimal default decisions can be used to calculate probabilities of default – an important input for risk management and pricing purposes. Numerical examples are provided, calibrated to US data. In a low interest rate environment, the credit-upgrade potential may *discourage* subprime borrowers from defaulting. However, default probabilities are highly sensitive to changes in interest rates and house prices. This provides a rational explanation for the prevalence of adjustable rate mortgages among subprime borrowers, and the subsequent large numbers of defaults, when interest rates rose and house prices declined.



## 1. Introduction

A great deal of press has been made of the subprime market crisis both in the US and internationally. Although many of the vectors by which contagion has spread through financial markets are to be found in the structured product market, or through the trading of credit derivatives between banks, at a grass roots level, the crisis has been caused by the default of households on their home loans. This paper focuses on the household's decision, asking the question: what motivates a household to default on its mortgage? To address this question, we focus on a real options style analysis of the household's decision. Such a model presumes that a household chooses to default or prepay their mortgage in such a way as to maximise their wealth.[1]

Given recent experiences with the securitisation of subprime mortgages through the use of collateralised debt obligations (CDOs) this analysis is particularly pertinent. One of the main advantages of the formation of CDOs is the ability to create some securities whose payoffs are relatively secure, even if the individual underlying securities have relatively high credit risk. This will only work well when the underlying securities' defaults are weakly correlated. As has been discovered in practice, mortgage defaults may not be so independent as one might hope. In particular, as interest rates change, whether due to changes in market rates, or the movement of hybrid adjustable rate mortgages from their fixed leg to their floating leg, a large number of households may simultaneously find themselves in a position where exercise of their default option is optimal. Correlation of default may be largely caused by the optimal use of the mortgage's embedded options, rather than idiosyncratic factors affecting individual households. Thus the considerations in this paper have very real impacts on risk management for investors in mortgage portfolios.

Thinking about the default decision using options is not entirely a new concept. Early work on pricing mortgages presumed that they can be valued as a financial derivative, whose underlying state variables are the interest rate and house price. Kau et al. (1987, 1993) outline the valuation of mortgage backed securities where the underlying securities are fixed rate and adjustable rate mortgages, respectively (or FRMs and ARMs as they are often abbreviated). In both cases, the household chooses to default or prepay their mortgage in a wealth-optimising fashion.

Of course, not all decisions to default or refinance are motivated by the level of house prices and interest rates. Many households, for example, choose to prepay their mortgages as a result of selling the house, which may be motivated by other economic or personal decisions. As a result, in considering the real options problem faced by a household, one should consider that a household may also exercise their options suboptimally. Dunn and McConnell (1982) initially used this approach to model prepayment, and Kau et al. (1992) show how it can be blended with optimal option use in the valuation of fixed rate mortgages. In our analysis, we approach the problem entirely from the household's perspective, to be

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[1] Or, since the loan is a liability to the household, to minimise the value of the loan.

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contrasted with the conventional pricing problem where we would be thinking about the value from the viewpoint of an investor in a mortgage backed security. This allows us to further explore the household's response to costs associated with default, whether financial or social. More importantly, it allows us to consider the effect of a subprime credit spread on the household's decision to default.

Some readers may question the validity of a real options type analysis when modelling the behaviour of households, many of whom may be considered to lack financial sophistication. However, recently, the availability of micro level data for individual mortgages has enabled econometricians to test whether households exercise their options optimally or not. Stanton (1995) examines the decision by FRM borrowers to refinance their loans, while Deng et al. (2000) and Calhoun and Deng (2002), examine FRM and ARM loans (respectively) providing evidence that empirically, households default and refinancing decisions seem to be strongly influenced by the moneyness of the relative options: households most often default when their house's value lies well below the present value of their future payments, and they refinance when the market value of their future payments is higher than the principal outstanding. As noted in Deng et al. (2000), although the real options problem faced by households is a complicated one, the conditions under which they should exercise their options can often be quite easy to determine, if the household can observe market prices for their mortgage.[2]

One critical difference between prime and subprime borrowers is the interest rate faced by the two types of borrowers. In general, subprime borrowers face higher rates, to compensate the lender for the credit risk involved in their loan. However, many subprime borrowers during a period of relatively rapid house price appreciation, such as during the early 2000s, would reason that the higher interest rate is an acceptable price to pay, since if house prices rise, they will accumulate sufficient equity in their property to refinance into a prime loan. Pennington-Cross and Chomsisengphet (2007) note that for many subprime borrowers, their home loan can also be a useful source of (relatively) low interest financing enabling them to consolidate their higher interest rate personal debt.

This paper uses a real options analysis based on the pricing methodologies mentioned above, to trace the optimal exercise frontier for mortgage default. This enables us to explain which combination of house prices and interest rates would trigger a household to want to default and, for situations where the household should not default immediately, the probability that they will default in the future. We compare and contrast traditional, amortising, mortgages, with the recently popular interest-only mortgages. Our findings demonstrate a strong conditionality of default: the level of house prices which will trigger default depends critically on the prevailing interest rate. This conditionality also depends on the structure of the loan, both in terms of the interest rate structure (ARM versus FRM) but also upon whether the loan is amortising or interest-only. We show that under some circumstances, interest-only loans may be *more* credit-worthy than amortising loans. We demonstrate the robustness of our results to the potential for suboptimal default or refinancing, as discussed above. We also consider the possibility that a household faces some stigma cost to defaulting, whether financial or non-financial, and show that this does not qualitatively change our findings.

The outline of the paper is as follows: Section 2 develops a model for house price and interest rate movements. Section 3 describes the options available to a household and how the optimal exercise of these can be determined to find the states in which the household will default (and the probability of reaching one of these states). Section 4 presents empirical estimates of the housing model in Section 2. Section 5 uses these estimates and the analysis from Section 3 to present numerical examples of household default behaviour. Lastly, Section 6 concludes.

## 2. A model for housing

Critical to understanding the relation between the structuring of residential mortgages and the optimal default behaviour of mortgagors is an understanding of the relation between interest rates and residential property prices. To examine that relation we assume a continuous time model for house prices ($H$) and interest rates, as suggested by Kau et al. (1993):

$$dH = (r + \xi_H)Hdt + \sigma_H H dW_H, \tag{1}$$

where $dW_H$ is a Brownian motion and $\xi_H$ and $\sigma_H$ are constants, reflecting the risk premium for house price growth and the volatility of house prices. $r$ is the instantaneous interest rate faced by households borrowing in the prime market. We assume that housing provides a continuous stream of rent, proportional to the house's value ($qHdt$) which is either consumed directly by the house's owner or earned as revenue by leasing the house. In the risk-neutral world, the process for the house's value is:

$$dH = (r - q)Hdt + \sigma_H H dW_H. \tag{2}$$

We assume a Cox–Ingersoll–Ross (CIR) process for the short interest rate:

$$dr = (\alpha - \beta r)dt + \sigma_r \sqrt{r} dW_r, \tag{3}$$

where $dW_r$ is a second Brownian motion, and $\alpha$, $\beta$ and $\sigma_r$ are constants. The correlation between $dW_r$ and $dW_H$ we define as $\rho$.

The CIR model for interest rates allows interest rates to vary over time, but features mean reversion: in the long term, interest rates will return to some (fixed) steady state. Interest rates can be correlated with house prices, reflecting demand side effects (the cost of financing for purchasers) and supply side effects (the cost of constructing new housing capital). The assumption that house prices follow a geometric Brownian motion is a simple one, and is a similar assumption to that commonly used in financial option pricing. However, taken in conjunction with a stochastic interest rates process, it provides plenty of traction for analysing option exercise by households.

When dealing with ARM loans, it is worth noting that the $T$ year zero coupon rate in this model is given by:

$$r(T) = \frac{-2\alpha}{\sigma_r^2 T}\log(A(T)) + \frac{r}{T}D(T), \tag{4}$$

where we define:

$$A(T) = \frac{2\gamma e^{\frac{(\beta+\gamma)T}{2}}}{(\beta+\gamma)(e^{\gamma T}-1)+2\gamma}$$
$$D(T) = \frac{2(e^{\gamma T}-1)}{(\beta+\gamma)(e^{\gamma T}-1)+2\gamma}$$
$$\gamma = \sqrt{\beta^2 + 2\sigma_r^2}.$$

If the actual underlying house price for a borrower was observable at regular intervals, the parameters of (1) and (3) could be estimated using Generalised Method of Moments (GMM) estimation, based upon the work of Chan et al. (1992). Note that this presumes that the risk-neutral process for interest rates is identical to the physical process. An alternative approach, which would obviate this assumption, would be to work with yield curve information

[2] An analogy can be made to regular financial option exercise. If one can observe the market price of a call or put option, exercise is optimal when the exercise value equals the option's value.