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The role of no-arbitrage on forecasting: Lessons from a parametric term structure model

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ABSTRACT

Parametric term structure models have been successfully applied to numerous problems in fixed income markets, including pricing, hedging, managing risk, as well as to the study of monetary policy implications. In turn, dynamic term structure models, equipped with stronger economic structure, have been mainly adopted to price derivatives and explain empirical stylized facts. In this paper, we combine flavors of those two classes of models to test whether no-arbitrage affects forecasting. We construct cross-sectional (allowing arbitrages) and arbitrage-free versions of a parametric polynomial model to analyze how well they predict out-of-sample interest rates. Based on US Treasury yield data, we find that no-arbitrage restrictions significantly improve forecasts. Arbitrage-free versions achieve overall smaller biases and root mean square errors for most maturities and forecasting horizons. Furthermore, a decomposition of forecasts into forward-rates and holding return premia indicates that the superior performance of no-arbitrage versions is due to a better identification of bond risk premium.

1. Introduction

Fixed income portfolio managers, central bankers, and market participants are in a continuous search for econometric models to better capture the evolution of interest rates. As the term structure of interest rates carries important information about monetary policy and market risk factors, those models might be seen as useful decision-orienting tools. In fact, in a quest to better understand the behavior of interest rates, a large literature on excess return predictability and interest rate forecasting has emerged.[1] In particular, some not intertemporally consistent while others impose no-arbitrage restrictions, and so far the importance of such restrictions on the forecasting context has not been established yet.

Testing the importance of no-arbitrage for interest rate forecasts should be relevant for at least two reasons. First, since imposing no-arbitrage implies stronger economic structure, testing how it will affect a model's ability to capture risk premium dynamics should be of direct concern to researchers. In principle, although we could expect that a more theoretically-sound model would better capture risk premia, only careful empirical analysis might manage to answer such question. On the other hand, from a practitioner's viewpoint, testing how no-arbitrage affects forecasting will objectively inform managers on whether it is worth to implement more complex interest rate models or not. Since latent factor models with no economic restrictions usually represent a simpler alternative to be implemented, if no-arbitrage restrictions do not aggregate practical gains, they do not necessarily have to be enforced.

In this paper, we address the above-mentioned points by testing how no-arbitrage restrictions affect the forecasting ability and risk premium structure of a parametric term structure model.[2] We argue that parametric models are particularly appropriate to test the effects of no-arbitrage on forecasting, since they keep a *fixed* factor-loading structure that is *independent* of the dynamics of underlying factors. This invariant loading structure implies that bond risk

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[1] Fama and Bliss (1987), Dai and Singleton (2002), Duffee (2002), and Cochrane and Piazzesi (2005) analyze the failure of the expectation hypothesis and the importance of time-varying risk premia. Diebold and Li (2006), and Bowsher and Meeks (2006) study different model specifications in a search for adequate forecasting candidates. Ang and Piazzesi (2003), Hordahl et al. (2006), and Favero et al. (2007) relate interest rates and macroeconomic variables through term structure models.

[2] In parametric term structure models, the term structure is a linear combination of predefined parametric functions, such as polynomials, exponentials, or trigonometric functions, among others. See, for instance, Nelson and Siegel (1987), and Svensson (1994), among others.

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premia relate to a common set of underlying factors (i.e. term structure movements) across different versions of the model. Based on this fixed set of factors, it should be possible to perform a careful analysis of how each model version and no-arbitrage restrictions affect risk premium.

We parameterize the term structure of interest rates as a linear combination of Legendre polynomials. This framework supports flexible dynamics for term structure factors, including versions that allow for arbitrage opportunities and others that are arbitrage-free. By focusing the analysis on three-factor models,[3] we compare a cross-sectional (CS) version, which allows for the existence of arbitrages, with two affine arbitrage-free versions, one Gaussian (AFG) and the other with one factor driving stochastic volatility (AFSV).

The CS polynomial version is similar to the exponential model adopted by Diebold and Li (2006) to forecast the US term structure of Treasury bonds, i.e. they are both parametric models that do not rule out arbitrages. In turn, the arbitrage-free versions of the Legendre model share many characteristics with the class of affine models proposed by Duffie and Kan (1996). No-arbitrage restrictions are imposed through the inclusion of conditionally deterministic factors of small magnitude that guarantee the existence of an equivalent martingale probability measure (Almeida, 2005). Each arbitrage-free version is implemented with six latent factors: three stochastic and three conditionally deterministic ones. Interestingly, by affecting the dynamics of the three basic stochastic factors ("level", "slope" and "curvature"), the conditionally deterministic factors directly affect the bond risk premium structure.

More general arbitrage-free versions of the polynomial model exist and could also be analyzed.[4] However, in an attempt to achieve more objectivity and transparency, a more concise analysis was favored, with choices of Gaussian (AFG) and stochastic volatility (AFSV) affine versions motivated by Dai and Singleton (2002), Duffee (2002), and Tang and Xia (2007). Duffee (2002) elects the three-factor affine Gaussian model as the best (within the affine family) to predict US bond excess returns. Dai and Singleton (2002) identify that the same Gaussian model correctly reproduces the failures of the expectation hypothesis documented by Fama and Bliss (1987) for US Treasury bonds. In contrast, Tang and Xia (2007) show that a three-factor affine model with one factor driving stochastic volatility generates bond risk premium patterns compatible with data from five major fixed income markets (Canada, Japan, UK, USA, and Germany). A key ingredient to all these findings is the flexible essentially affine parameterization of the market prices of risk (Duffee, 2002), which we also adopt in our work.

Based on monthly US zero-coupon Treasury data, we analyze the out-of-sample behavior of the three proposed versions under different forecasting horizons (1-month, 6-month, and 12-month). Forecasting results indicate that dynamic arbitrage-free versions of the model achieve overall lower bias and root mean square errors for most maturities, with stronger results holding for longer forecasting horizons. Diebold and Mariano's (1995) tests confirm the statistical significance of the obtained results.

In order to analyze the effects of no-arbitrage on the risk premium structure, we decompose yield forecasts into forward rates and risk premium components. The decomposition allows us to identify that the superior forecasting performance of arbitrage-free versions is primarily due to a better identification of bond risk premium dynamics. This result represents an important effort in the direction of understanding *how* no-arbitrage affects forecasting. It also indicates that further analysis with other classes of parametric models should be seriously considered.

Related works include the papers by Duffee (2002), Ang and Piazzesi (2003), Favero et al. (2007), and Christensen et al. (2007). Duffee (2002) tests the ability of affine models on forecasts of interest rates, concluding that completely affine models fail to reproduce the stylized facts of US term structure, while essentially affine models do a better job due to a richer risk premium structure. While Duffee (2002) analyzes how different market prices of risk specifications affect forecasting in *arbitrage-free models*, we study how no-arbitrage affects forecasting, which means including models that allow for arbitrages in our analysis.

Ang and Piazzesi (2003) show that imposing no-arbitrage restrictions to a VAR model with macroeconomic variables improves its forecasting ability. Similarly, Favero et al. (2007) test how macroeconomic variables and no-arbitrage restrictions affect interest rate forecasting, finding that no-arbitrage models, when supplemented with macro data, are more effective in forecasting. Both papers model factor dynamics with a Gaussian VAR structure, while we include stochastic volatility in our analysis, finding it to be relevant to improve forecasting. In addition, both allow for changes in term structure loadings when comparing no-arbitrage models to models that allow for arbitrages. Those changes in factors and bond risk premia make it harder to isolate the pure effects of no-arbitrage on forecasting. In contrast, the parametric polynomial term structure model adopted in our work avoids this issue due to its fixed factor loading structure.

Christensen et al. (2007) obtain a Gaussian arbitrage-free version of the parametric exponential model proposed by Diebold and Li (2006). They empirically test their arbitrage-free version and identify that it offers predictive gains for moderate to long maturities and forecasting horizons. Although in this case they keep a fixed factor loading structure as we do, there are interesting differences between the two papers. First, the two papers analyze distinct parametric families, each offering interesting insights. Second, the technique used to derive arbitrage-free versions is quite distinct. While we base our derivations on Filipovic's (2001) consistency work, which is not attached to the class of affine models, they make use of Duffie and Kan's (1996) arguments, which are valid only under affine models. Third, they present a Gaussian arbitrage-free version while we also include the important case where volatility is stochastic. Finally, in addition to the forecasting analysis, we propose a careful analysis of the risk premium structure, which should be particularly interesting for portfolio managers and risk managers, as a complementing tool.

Our results should be important to managers and practitioners in general. They suggest it should be worth constructing arbitrage-free versions of other parametric models to test their performances as practical forecasting/hedging tools. The techniques adopted to construct arbitrage-free versions of the polynomial model can be found in Filipovic (2001) and can be readily applied to other parametric families, such as variations of the Nelson and Siegel (1987) model, the Svensson (1994) model,[5] and spline models with fixed knots, among others.

We provide evidence that no-arbitrage restrictions improve interest rate forecasting for a class of parametric models. However, what is the extent of this conclusion? Our results when coupled

[3] Litterman and Scheinkman (1991) show that most of the variability of the US term structure of Treasury bonds can be captured by three factors: level, slope and curvature. Many subsequent more recent works have confirmed their findings. An exception is Cochrane and Piazzesi (2005) who find that a fourth latent factor improves forecasting ability.

[4] For instance, versions with more than one factor driving stochastic volatility within the affine family, or even models with a non-affine diffusion structure. See, for example, Almeida (2005).

[5] Filipovic (2001) shows that there is no non-trivial arbitrage-free version of the original Nelson and Siegel (1987) model. Nevertheless, it is possible to construct arbitrage-free versions of variations of the Nelson and Siegel (1987) and Svensson (1994) models, as shown for instance, by Sharef and Filipovic (2004), and Christensen et al. (2007).