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Conditions on option prices for absence of arbitrage and exact calibration

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Abstract

Under the assumption of absence of arbitrage, European option quotes on a given asset must satisfy well-known inequalities, which have been described in the landmark paper of Merton [Merton, R., 1973. Theory of rational option pricing. Bell Journal of Economics and Management Science 4 (1), 141–183]. If we further assume that there is no interest rate volatility and that the underlying asset continuously pays deterministic dividends, cross-maturity inequalities must also be satisfied by the bid and ask option prices.

In this paper, we show that there exists an arbitrage-free model, which is consistent with the option quotes, if these inequalities are satisfied. One implication is that all static arbitrage strategies are linear combinations, with positive weights, of those described here. We also characterize admissible default probabilities for models which are consistent with given option quotes.

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1. Introduction

Since the 1987 crash, which showed some of the shortcomings of the Black–Scholes model (see Black and Scholes, 1973), numerous models have been proposed to better fit European option quotes. However, those who claim this fitting to be exact are rare.

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Among them we should cite local volatility models, described in Derman and Kani (1994) and Dupire (1994) as well as local Lévy models introduced in Carr et al. (2004). But these models assume that European options of all strikes and maturities are traded, which leads in practice to an interpolation of the market option prices. And, to the best of our knowledge, no rigorous algorithm has been described so far to calibrate these models exactly in practice. For instance, Derman and Kani (1994) advise to ignore some quotes when the latters produce arbitrage in their model.

One objective of the present paper is to give conditions on quoted European option prices which allow exact calibration by an arbitrage-free model. Notice that these conditions will be necessary for the absence of arbitrage.

This part of the paper has mainly been inspired by Carr and Madan (2005) and we will generalize their results by allowing:

- the underlying asset to pay deterministic dividends continuously,
- deterministic interest rates,
- the options to have any strike or maturity and to be in finite number,
- the options to have different bid and ask prices.

Two recent working papers, Buehler (2004) and Davis and Hobson (2004), have the same goal and a similar treatment. Nevertheless Buehler (2004) does not provide arbitrage strategies which are related to conditions on option prices but instead describes criteria to choose a more realistic model in the subset of calibrated arbitrage-free Markov chain models, as does Cousot (2004). Furthermore, both preprints do not allow different bid and ask prices.

We should also acknowledge the work of Laurent and Leisen (1998) which obtained similar results in spirit by extending the concept of Arrow Debreu securities and by constructing a Markov chain model fitting the market quotes. Nevertheless our treatment is perhaps more rigourous since we attach a great importance to justifying the existence of what would be in their case transition matrices which give rise to martingales. Moreover, as the two working papers cited above, their framework does not encompass the difference between bid and ask prices.

Another objective of the present paper is to characterize the default probabilities which are admissible for the stock among the arbitrage-free models which are calibrated to a given set of option prices. Indeed, with the emergence of credit derivative products such as Credit Default Swaps (CDS), it is more and more desirable for an equity model to be calibrated to both option prices and default probabilities. (See Atlan and Leblanc (2005), Carr and Linetsky (2006) and the references herein for examples of models which attempt to capture both). Nevertheless, to the best of our knowledge, admissible default probabilities have never been characterized.

The structure of the paper is given as follows. Section 2 specifies our assumptions and introduces the definitions of calendar vertical spreads and calendar butterfly spreads. Section 3 describes necessary conditions on option quotes under the assumption of no arbitrage and also discusses briefly why these conditions are not sufficient without further assumptions. Section 4 shows that these conditions are not only necessary but also sufficient for the existence of a calibrated arbitrage-free model. Section 5 describes the set of admissible default probabilities of such a model. Section 6 concludes.