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Covariance complexity and rates of return on assets

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Abstract

This paper considers the estimation of the expected rate of return on a set of risky assets. The approach to estimation focuses on the covariance matrix for the returns. The structure in the covariance matrix determines shared information which is useful in estimating the mean return for each asset. An empirical Bayes estimator is developed using the covariance structure of the returns distribution. The estimator is an improvement on the maximum likelihood and Bayes–Stein estimators in terms of mean squared error. The effect of reduced estimation error on accumulated wealth is analyzed for the portfolio choice model with constant relative risk aversion utility.

JEL classification: C11; C13; C51; G11

Keywords: Empirical Bayes estimation; Rates of return; Covariance complexity; Wealth loss

1. Introduction

The allocation of investment capital to risky and risk free opportunities is a fundamental problem in portfolio theory. A basic input to the investment decision is the distribution of future returns on securities. The prediction of future securities returns is based on price information available at the time of the decision. Prediction errors can have a large negative impact on portfolio choice and the resulting accumulation of wealth (Loffler, 2003).

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Errors in the prediction of mean returns are particularly damaging to wealth accumulation (Kallberg and Ziemba, 1981, 1984; Chopra and Ziemba, 1993).

A standard approach to the trading prices of securities is to consider the rate of return as a linear model (Connor and Korajczyk, 1995). The rate of return is defined over a period (Sharpe, 1964; Ross, 1976) or instantaneously (Merton, 1992). The rate depends on a variety of factors, observed and unobserved. The returns on different securities are linked through the factors. When the model is underspecified, there are unobserved factors and the prices are mis-specified (MacKinlay and Pastor, 2000). Because of the link to common factors, the mispricing is incorporated into the returns covariance matrix.

The parameters in the linear model for returns may be random variables, which requires a hierarchy of stochastic equations. The random coefficients in the stochastic equations are random parameters in the securities price distributions, and the hierarchy generates a Bayesian model for price distributions. This is an alternative approach to the missing or unobserved factors. In many Bayesian approaches, a noninformative or diffuse prior on parameters in the price distribution is postulated (Klein and Bawa, 1976; Pastor and Stambaugh, 1999). Alternatively, a conjugate prior can used so that the posterior is tractable. In the Gaussian case, this provides a framework for understanding the mechanism for sharing common information (Jones and Shanken, 2003).

In the context of the Bayes model, the optimal estimates for the expected rates of return given price information to date are the posterior means, which depend on the covariance of returns. This implies a sharing of information on asset returns to yield improved estimates. As well, the decomposition of the covariance into a prior component (common) and a conditional component (specific) isolates the mis-specification effect.

One difficulty with the Bayes approach is that the parameters in the prior distribution are unknown. There are theories which help define the prior. However, the data on returns contains information about the prior, and in the Gaussian case the prior can be filtered from the data. In that way, an empirical Bayes estimate for the posterior mean is developed (Efron and Morris, 1972; Frost and Savarino, 1986). In the dynamic model format, the observed prices are points on a trajectory and the movement of prices provides the necessary information to estimate the conditional (first order) and prior (second order) parameters in the hierarchial linear model.

The impact of modeling and estimation errors on forecasts for securities prices and the resulting effect on portfolio decisions and capital accumulation have been considered in several studies. Pastor and Stambaugh (1999) conclude that model error is less important than estimation error; see also Kallberg and Ziemba (1981) who conclude the same. With regard to estimation error, alternative estimates for the mean return have been considered in a long series of asset prices (Grauer and Hakansson, 1995), with improved results from shrinkage (Stein) estimators. MacKinlay and Pastor (2000) use a restriction, which incorporates the mean return in the covariance of returns, to calculate an estimate of expected returns which is superior to the shrinkage estimator. The results are empirical rather than theoretical, and the structure and dynamics of price distributions is not clear.

This paper further explores the connection between the expected rates of return on a set of assets and the covariance of returns on those assets. The setting is similar to MacKinlay and Pastor (2000), with unobserved factors linking the means to the covariance. The latent factors define a prior on mean returns, and Bayes theorem determines a posterior mean which depends on the covariance. Information on the relationship between asset returns is contained in the covariance, and an entropy measure called covariance complexity