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Pricing options on scenario trees

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Abstract

We examine valuation procedures that can be applied to incorporate options in scenario-based portfolio optimization models. Stochastic programming models use discrete scenarios to represent the stochastic evolution of asset prices. At issue is the adoption of suitable procedures to price options on the basis of the postulated discrete distributions of asset prices so as to ensure internally consistent portfolio optimization models. We adapt and implement two methods to price European options in accordance with discrete distributions represented by scenario trees and assess their performance with numerical tests. We consider features of option prices that are observed in practice. We find that asymmetries and/or leptokurtic features in the distribution of the underlying materially affect option prices; we quantify the impact of higher moments (skewness and excess kurtosis) on option prices. We demonstrate through empirical tests using market prices of the S&P500 stock index and options on the index that the proposed procedures consistently approximate the observed prices of options under different market regimes, especially for deep out-of-the-money options.

JEL classification: G12; G13; C6; C00

Keywords: Stochastic programming; Pricing under discrete distributions; European options; Risk-neutral probabilities; Effect of higher order moments

1. Introduction

Stochastic programs provide an effective framework for modelling diverse financial management problems. The deployment of stochastic programming models in practice is made possible due to flexible modelling systems, algorithmic developments for large-scale stochastic programs and advancements of computing technology. As a result, stochastic programs are increasingly gaining acceptance as viable tools for addressing diverse financial planning problems under uncertainty. Recent advancements of models and applications of stochastic programs for portfolio management and related problems are documented, for example, in Ziemba and Mulvey (1998), Dupačová et al. (2002), Ziemba (2003), Zenios and Ziemba (2006, 2007), Vladimirou (2007) and Zenios (in press).

Stochastic programs are gaining popularity because of their flexibility and several advantageous features. Multistage stochastic programs provide an effective basis to model dynamic portfolio management problems. They can incorporate practical considerations such as portfolios of many assets, transaction costs, liquidity, trading and turnover constraints, limits on holdings in individual assets or groups, as well as additional managerial and regulatory requirements that can be modelled as linear constraints on the decision variables. Moreover, stochastic programs can accommodate different objective functions to represent the decision maker's risk bearing attitude and the primary decision goals. These features provide substantial leeway in modelling practical financial planning problems.

In dynamic stochastic programs uncertainty in input parameters is flexibly modelled in terms of discrete distributions. The discrete distributions can capture the joint co-variation of the random variables and are represented by means of a scenario tree that depicts the progressive evolution of the random variables. The models are not

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restricted by rigid distributional assumptions. Asymmetric and heavy-tailed distributions of random variables, that are often implied by financial times series, can be accommodated in scenario-based models.

These flexibilities of stochastic programs make them suitable tools for risk management applications. It is natural to consider, in the context of stochastic programs, appropriate instruments that can serve the risk management function. Derivatives are well-suited for this purpose. By construction, options have asymmetric payoffs that can cover against adverse price movements of the underlying security. Combinations of options can help shape a desired payoff profile over the range of possible movements in the price of the underlyings. Consequently, the incorporation of options in portfolio optimization models should lead to improved risk management tools.

It is not our intention to provide a comprehensive review of the extensive literature on option pricing; we only cite some representative works.

In their seminal work, Black and Scholes (1973) derived explicit pricing formulas for both call and put options based on the assumption that stock prices follow a geometric Brownian motion. Several empirical studies (e.g. Rubinstein, 1985; Rubinstein, 1994) showed that the B–S model misprices deep out-of-the money options. The volatility estimates in the Black–Scholes formula, implied by market prices of options and their underlying securities, differ across exercise prices and maturities and form "smile" patterns, violating the constant volatility assumption. Empirical estimates of the risk-neutral probability density of asset returns reveal (often negatively) skewed and leptokurtic distributions, in contrast to the log-normal distribution assumed in the Black–Scholes model.

Dempster and Hutton (1999), Dempster et al. (1998) and Dempster and Richards (2000) developed, implemented and tested procedures based on linear programming for pricing American and exotic options in the Black–Scholes framework. Their procedures exploit the problem structure in specializing operations of the revised simplex algorithm to solve efficiently finite difference approximations of the Black–Scholes differential equation.

Another option pricing approach infers the risk-neutral distribution for the price of the underlying security. The observed financial time series and risk premia imply a risk-neutral probability measure that can be used to price any derivative as the expected discounted value of its future payoff. While the physical (empirical, subjective) and risk-neutral probability measures are related, they are identical only in the case of zero premia on all relevant risks factors. A fundamental theorem of asset pricing states that in the absence of arbitrage there exists some pricing kernel that can reconcile the two measures. This result prompted the studies by Aït-Sahalia and Lo (2000), Jackwerth (2000) and Rosenberg and Engle (2002) regarding the characteristics of the pricing kernel.

An alternative approach to deal with nonconstant volatility was proposed by Rubinstein (1994) and Jackwerth and Rubinstein (1996), and in a series of papers by Derman and Kani (1994), Derman et al. (1996) and Dupire (1994). Instead of imposing a parametric function for volatility, they approximate the structure of asset prices with binomial or trinomial lattices which are calibrated on the basis of market prices. Rubinstein (1994) showed how to compute the implied distribution using quadratic programming; Jackwerth and Rubinstein (1996) generalized this approach using nonlinear programming to minimize four different objective functions.

To incorporate options in scenario-based portfolio optimization models the options must be priced consistently with the price scenarios for the underlying securities. The following key issue needs to be resolved. Common option pricing methods typically rely on specific assumptions regarding the stochastic process of the underlying security's price. On the other hand, stochastic programming models afford the flexibility to cope with general discrete distributions. It is often desirable, and even necessary, to capture in the scenario sets skewness and excess kurtosis features that are often observed in market data. Hence, the postulated scenarios for the asset prices do not necessarily conform to distributional assumptions on which popular option valuation methods are based. As a result, these option pricing methods cannot be employed if the options are to be incorporated in stochastic programming models that use a different representation of uncertainty for the prices of the underlying assets. Otherwise, the models can give rise to arbitrage opportunities and imply spurious profits. Appropriate option pricing procedures must be adopted to ensure internal consistency of the stochastic programming model.

This paper aims to adapt, implement and empirically validate appropriate methods to price options in accordance with discrete scenario sets of asset prices. We confine our attention to European options and propose two procedures to price the options consistently with the discrete distributions of a scenario tree. The adopted pricing procedures account for the statistical characteristics of asset returns (including skewness and kurtosis) as reflected in their empirical distributions. A key concern is to ensure that the scenarios of asset prices, in conjunction with the resulting option prices, satisfy the fundamental no-arbitrage condition.

In the first approach, we determine an equivalent risk-neutral probability measure from the physical probability measure that is associated with the postulated price outcomes on the scenario tree. For European options, the critical input for valuation is the distribution of the underlying asset's price (and, consequently, the option payoff) at the time of the option's maturity. In our construction, the discrete support of this distribution is represented by a corresponding subset of nodes of the scenario tree. Once the risk-neutral probabilities of these nodes (states) are determined, pricing the options is straightforward. The price of a European option is the expectation of its discounted payoff. The expectation is taken, under the risk-neutral probability measure, over the nodes of the scenario