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Selecting copulas for risk management ☆

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Abstract

Copulas offer financial risk managers a powerful tool to model the dependence between the different elements of a portfolio and are preferable to the traditional, correlation-based approach. In this paper, we show the importance of selecting an accurate copula for risk management. We extend standard goodness-of-fit tests to copulas. Contrary to existing, indirect tests, these tests can be applied to any copula of any dimension and are based on a direct comparison of a given copula with observed data. For a portfolio consisting of stocks, bonds and real estate, these tests provide clear evidence in favor of the Student's *t* copula, and reject both the correlation-based Gaussian copula and the extreme value-based Gumbel copula. In comparison with the Student's *t* copula, we find that the Gaussian copula underestimates the probability of joint extreme downward movements, while the Gumbel copula overestimates this risk. Similarly we establish that the Gaussian copula is too optimistic on diversification benefits, while the Gumbel copula is too pessimistic. Moreover, these differences are significant.

JEL classification: G11; C12; C14

Keywords: Financial dependence; Risk management; Copulas; Distributional tests; Tail dependence

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1. Introduction

Modelling dependence is of key importance to portfolio construction and risk management. An inappropriate model for dependence can lead to suboptimal portfolios and inaccurate assessments of risk exposures. Traditionally, correlation is used to describe dependence between random variables, but recent studies have ascertained the superiority of copulas to model dependence, as they offer much more flexibility than the correlation approach (see e.g., Embrechts et al., 2002). An important reason to consider other copulas than the correlation-implied Gaussian copula is the failure of the correlation approach to capture dependence between extreme events, as shown by Longin and Solnik (2001), Bae et al. (2003) and Hartmann et al. (2004). However, up to now no consensus has been reached on which copula to use in specific applications or on how to test the accuracy of a specific copula.

In this paper, we propose an approach to evaluate copulas and investigate the importance of accurate copula selection. Generally, theory offers little guidance in choosing a copula, making the selection an empirical issue. Since a copula is equivalent to a distribution function, we discuss how traditional goodness-of-fit tests such as the Kolmogorov–Smirnov test and the Anderson–Darling test can be applied. These tests are based on a direct comparison of the dependence implied by the copula with the dependence observed in the data. These direct tests of the fit of a copula have several advantages over alternative approaches proposed in the literature. First, they are applicable to any copula, not only to the Student's t and Gaussian copulas. Second, they can be used for copulas of any dimension, not only for bivariate copulas. Third, they indicate whether a copula captures the observed dependence accurately, and not only whether it can be rejected against another specific copula. Finally, they take the complete dependence into account, contrary to selection procedures that consider only part of the dependence pattern (i.e., dependence of extreme observations).

To determine the importance of selecting the right copula for risk management, we consider a portfolio of stocks, bonds and real estate. As investors are generally averse to downside risk, a copula should capture both the risk of joint downward movements of asset prices, and the diversification opportunities that assets offer. This is particularly relevant in the case of stocks, bonds and real estate, as a proper allocation over these assets should lead to the main risk reduction in investments. Therefore, we test the Gaussian, the Student's t and the Gumbel copulas to model the dependence of the daily returns on indexes that approximate these three asset classes. The Gaussian copula is the traditional candidate for modelling dependence. The Gumbel copula is directly related to multivariate extensions of extreme value theory, which has gained popularity in risk management over the last decade (see e.g., Longin, 1996). The Student's t copula can be seen as a compromise, because it can capture dependence in the tails without giving up flexibility to model dependence in the center.

In our application, the Student's t copula passes the tests with success, but both the Gaussian and Gumbel copulas are rejected. To stress the economic importance of copulas in this application, we show that the different copulas lead to significantly different assessments both of the risk of downward movements and of diversification benefits. We examine the risk of downward movements by stress tests. The Student's t copula leads to probabilities that do not differ significantly from the empirical copula. On the contrary, the Gaussian copula significantly underestimates the risk of joint downward movements