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ScienceDirect
Journal of Banking & Finance 31 (2007) 3125–3144
Journal of BANKING & FINANCE
www.elsevier.com/locate/jbf

# Yield-factor volatility models

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Received 6 December 2005; accepted 15 November 2006
Available online 24 January 2007

**Abstract**

The term structure of interest rates is often summarized using a handful of yield factors that capture shifts in the shape of the yield curve. In this paper, we develop a comprehensive model for volatility dynamics in the level, slope, and curvature of the yield curve that simultaneously includes level and GARCH effects along with regime shifts. We show that the level of the short rate is useful in modeling the volatility of the three yield factors and that there are significant GARCH effects present even after including a level effect. Further, we find that allowing for regime shifts in the factor volatilities dramatically improves the model's fit and strengthens the level effect. We also show that a regime-switching model with level and GARCH effects provides the best out-of-sample forecasting performance of yield volatility. We argue that the auxiliary models often used to estimate term structure models with simulation-based estimation techniques should be consistent with the main features of the yield curve that are identified by our model.


*JEL classification:* E43; C32; C51

*Keywords:* Yields; GARCH; Level effect; Regime shifts

## 1. Introduction

The term structure of interest rates is often summarized using a handful of yield factors that capture shifts in the shape of the yield curve, i.e., changes in the overall level, slope,

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0378-4266/$ - see front matter 
doi:10.1016/j.jbankfin.2006.11.016

and curvature of the yield curve (Litterman and Scheinkman, 1991). This factor decomposition provides a parsimonious representation of the term structure and is extensively used in fixed-income derivative pricing (Driessen et al., 2003), to model the linkages between interest rates and macroeconomic variables (Ang and Piazzesi, 2003), or to estimate term structure models (Brandt and Chapman, 2003). Despite the wide application of yield factors in financial economics, very little is known about their volatility dynamics. In this paper, we study the dynamics of yield-factor conditional volatility.

The key features of our econometric model are inspired by early models of the short-term interest rate. The main conclusion from this literature is that a level effect, in which the volatility is a positive function of the level of interest rates, GARCH effects, and regime shifts are required to adequately model the short-rate volatility. The dependence of interest rate volatility on the level of the short rate is systematically studied in Chan et al. (1992, hereafter CKLS). The first model that combines both level and GARCH effects for the short-rate volatility is proposed in Longstaff and Schwartz (1992). Further, Brenner et al. (1996) show that models that include both GARCH and level effects are better able to predict volatility than models that only include one of these effects. Gray (1996) extends the GARCH-level model to allow for multiple regimes in short-rate volatility and concludes that all three effects are needed to adequately model interest rate volatility.

Less research has been devoted to understanding the joint-dynamics of the level, slope, and curvature of the yield curve, which we call the yield factors. Pérignon and Villa (2006) show that the volatility of the yield factors is the primary source of time variation in the covariance matrix of interest rates. The role of conditional heteroscedasticity in the dynamics of the volatility of the yield factors has been highlighted by Christiansen and Lund (2002) and Christiansen (2004). The empirical challenge of estimating a level effect in a multi-factor framework has been tackled by Boudoukh et al. (1998), Brandt and Chapman (2003) and Christiansen (2005a). Another strand of research examines the role of regime shifts in the dynamics of yield-factor volatilities. Using international data, Kugler (1996) and Ang and Bekaert (2002a,b) estimate a two-state regime-switching VAR model of the level and the slope factors with a constant covariance matrix in each regime (i.e., without any level or GARCH effects). Christiansen (2004) extends the latter approach by fitting a two-state regime-switching ARCH model to the US level and the slope factors. A broad conclusion of this research is that regime shifts are a central feature of yield-factor volatilities.

Our primary contribution is the development of a comprehensive model for yield factors that simultaneously includes level and GARCH effects along with regime shifts in the factor volatilities. This model contrasts with comprehensive univariate short-rate models (Gray, 1996; Smith, 2002) and with bi- or tri-variate yield-factor models accounting for one or two of the above volatility features (Christiansen and Lund, 2002; Christiansen, 2004; Christiansen, 2005a). To the best of our knowledge, our multivariate model of the level, slope, and curvature of the yield curve is the first one to jointly include all these volatility features.

We model the variance of the three yield factors as a function of the short term interest rate for two reasons. First, empirically, the volatility of all three yield factors tends to be higher when short-term interest rates are higher. Second, we demonstrate theoretically that within the class of affine term structure models in which the volatility of all state variables is determined by a single state variable, i.e., the $\mathbb{A}_1(3)$ class in the terminology of Dai and Singleton (2000), both the level of the short-term interest rate and the variance of all three