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Model-free hedge ratios and scale-invariant models

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Abstract

A price process is scale-invariant if and only if the returns distribution is independent of the price measurement scale. We show that most stochastic processes used for pricing options on financial assets have this property and that many models not previously recognised as scale-invariant are indeed so. We also prove that price hedge ratios for a wide class of contingent claims under a wide class of pricing models are model-free. In particular, previous results on model-free price hedge ratios of vanilla options based on scale-invariant models are extended to any contingent claim with homogeneous pay-off, including complex, path-dependent options. However, model-free hedge ratios only have the minimum variance property in scale-invariant stochastic volatility models when price–volatility correlation is zero. In other stochastic volatility models and in scale-invariant local volatility models, model-free hedge ratios are not minimum variance ratios and our empirical results demonstrate that they are less efficient than minimum variance hedge ratios.

JEL classification: G13; C14

Keywords: Scale invariance; Model-free; Hedging; Minimum variance; Stochastic volatility; Local volatility

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1. Introduction

Do different option pricing models yield different hedge ratios? This important question is related to model error in option pricing models, an issue that has been addressed by Derman (1996), Green and Figlewski (1999), Cont (2006), Psychoyios and Skiadopoulos (2006) and others. Another challenging question, related to work by Bakshi et al. (1997, 2000) and Lee (2001), is whether minimum variance hedge ratios perform better than standard price hedge ratios in dynamic hedging within a stochastic volatility setting. This paper pursues the answer to these two questions by focussing on scale-invariant models and proving four main results.

A multitude of models for option pricing have been developed in recent years and the academic literature is enormous (see Jackwerth, 1999; Skiadopoulos, 2001; Bates, 2003; Psychoyios et al., 2003; Cont and Tankov, 2004, for comprehensive reviews). However, our first result implies that the vast majority of models share the common property of being scale-invariant. A price process is scale-invariant if and only if the asset price returns distribution is independent of the price measurement scale. The first result allows models to be classified as scale-invariant or otherwise without deriving the returns density. This is important because the returns distribution for many models is not known in analytic form. Thus it broadens the scale-invariant class to encompass models that have not previously been acknowledged as scale-invariant.

Two further results will prove that the price hedge ratios of virtually any claim are model-free, and any difference between the empirically observed hedge ratios can only be attributed to a different quality of the models' fit to market data. More precisely, the standard delta, gamma and higher order price hedge ratios are model-free in the class of scale-invariant models provided only that the claim's expiry pay-off is homogeneous of some degree in the price, strike and any other claim characteristic in the price dimension. Almost all claims in current use have such a homogeneity property.

Vanilla options (i.e. standard European and American calls and puts) have expiry pay-offs that are homogeneous of degree one in the underlying price and strike. Merton (1973) showed that when such options are priced under a scale-invariant process their prices at any time prior to expiry are also homogeneous of degree one. Our second result extends this property to other claims with homogeneous pay-offs: Suppose the expiry pay-off of a claim is homogeneous of degree k in the underlying price, strike and every other parameter in the price dimension (e.g. a barrier). Then, when priced under a scale-invariant process, the price at any time prior to expiry of the claim is also homogeneous of degree k. In other words, the prices of most path-dependent options, such as barriers, Asians, lookbacks and forward starts, and the prices of options with pay-offs that are homogeneous of degree $k \neq 1$ such as binary options and power options, at any time prior to expiry, have the same degree of homogeneity as their pay-off functions when they are priced under a scale-invariant process.

Bates (2005) proved that if an option *price* is homogeneous of degree one in the underlying price and strike then its standard delta and gamma are model-free in the class of scale-invariant processes. Our third result extends this model-free property: to options with *pay-off* functions that are homogeneous of *any* degree k in the price dimension, to higher-order price hedge ratios, and to include other characteristics in the price dimension, such as barriers.

Our fourth result is related to minimum variance hedging. The minimum variance (MV) hedge ratio is that ratio which minimizes the variance of the hedged portfolio. See Bakshi