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A linearly implicit predictor–corrector scheme for pricing American options using a penalty method approach

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Abstract

Pricing of an American option is complicated since at each time we have to determine not only the option value but also whether or not it should be exercised (early exercise constraint). This makes the valuation of an American option a free boundary problem. Typically at each time there is a particular value of the asset, which marks the boundary between two regions: to one side one should hold the option and to other side one should exercise it. Assuming that investors act optimally, the value of an American option cannot fall below the value that would be obtained if it were exercised early. Effectively, this means that the American option early exercise feature transforms the original linear pricing partial differential equation into a nonlinear one. We consider a penalty method approach in which the free and moving boundary is removed by adding a small and continuous penalty term to the Black–Scholes equation; consequently,the problem can be solved on a fixed domain. Analytical solutions of the Black–Scholes model of American option problems are seldom available and hence such derivatives must be priced by stable and efficient numerical techniques. Standard numerical methods involve the need to solve a system of nonlinear equations, evolving from the finite difference discretization of the nonlinear Black–Scholes model, at each time step by a Newton-type iterative procedure. We implement a novel linearly implicit scheme by treating the nonlinear

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penalty term explicitly, while maintaining superior accuracy and stability properties compared to the well-known θ-methods.

JEL classification: C63; G10

Keywords: Black–Scholes equation; Method of lines; Predictor–corrector methods; American options; Penalty method

1. Introduction

The development of modern option pricing began with the publication of the Black–Scholes option pricing formula in 1973. Black and Scholes (1973) and Merton (1973) gave a derivation of a model equation to compute the value of a European option. This equation has had such financial impact that Robert Merton and Myron Scholes shared the 1997 Nobel Prize for economics (Fischer Black having died in 1995). The Black–Scholes formula computes the value of a European option based on the underlying asset, strike price, volatility of the asset, and the time until the option expires; see, for example, Hull (1997), El Karoui et al. (1998), Little and Pant (2000), Stampfli and Goodman (2001) and references therein. The European option can be exercised only at expiry date whereas an American option has the additional feature that exercise is permitted at any time during the life of the option. Therefore, pricing of an American option is more complicated since at each time we have to determine not only the option value but also whether or not it should be exercised (early exercise constraint). This makes the valuation of an American option a free boundary problem. Typically at each time there is a particular value of the asset, which marks the boundary between two regions: to one side one should hold the option and to other side one should exercise it. Assuming that investors act optimally, the value of an American option cannot fall below the value that would be obtained if it were exercised early. Effectively, this means that the American option early exercise feature transforms the original linear European pricing equation into a nonlinear Partial Differential Equation (PDE). We consider an extended Black–Scholes model with a nonlinear penalty source term. This term allows the applicability of Black–Scholes model beyond the basic European option. In the penalty approach, the free and moving boundary is removed by adding a small and continuous penalty term to the Black–Scholes equation; consequently, the problem can be solved on a fixed domain. Analytical solutions of the Black–Scholes model of American option problems are seldom available and hence such derivatives must be priced by stable and efficient numerical techniques.

The penalty method was introduced by Zvan et al. (1998) for American options with stochastic volatility by adding a source term to the discrete equation. Nielsen et al. (2002) presented a refinement of their work by adding a penalty term to the continuous equation and illustrated the performance of various numerical schemes. Specifically, an explicit, a semi-implicit, and a fully implicit method were analyzed