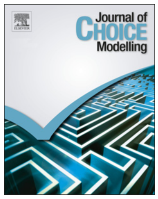
The Journal of Choice Modelling 10 (2014) 60–68

Contents lists available at ScienceDirect

The Journal of Choice Modelling

journal homepage: www.elsevier.com/locate/jocm



Best-Worst Scaling in analytical closed-form solution

Stan Lipovetsky *, Michael Conklin

GfK Custom Research North America, 8401 Golden Valley Rd., Minneapolis, MN 55427, USA

ARTICLE INFO

Article history:
Received 16 July 2013
Received in revised form
10 February 2014
Accepted 11 February 2014
Available online 14 March 2014

Keywords:
Best-Worst Scaling
MaxDiff
MNL
Utility
Choice probability
Marketing research

ABSTRACT

Best-Worst Scaling (BWS), sometimes also called Maximum Difference (MaxDiff), is a discrete choice modeling method widely used for finding utilities and choice probabilities among multiple alternatives. It can be seen as an extension of the paired comparison techniques for the simultaneous presentation of several items together to respondents. A respondent identifies the best and the worst ones and estimation of utilities is performed using a multinomial-logit (MNL) model in numerical nonlinear estimations. The main contribution of this paper consists in finding an analytical closed-form solution producing an approximation of the results for utilities and choice probabilities that are obtained using MNL models. The analytical formulae permit the inference of the characteristics of the model's quality, including standard errors of the utilities and choice probabilities, the residual deviance and pseudo-R^2. This approach enriches the BWS methods and is useful for theoretical descriptions and practical applications.

1. Introduction

Best-Worst Scaling (BWS), sometimes also called Maximum Difference (or MaxDiff) is a contemporary method for the prioritization of items proposed by Louviere (1991, 1993), and developed and applied in numerous works (for instance, Finn and Louviere, 1992; Louviere et al., 2000, 2008, 2013; Marley and Louviere, 2005; Bacon et al., 2007, 2008; Louviere, 2013; Flynn et al., 2013; Hess and Daly, 2013; Frischknecht et al., in press). BWS is based, on the one hand, on extended scaling methods known in Thurstone, Bradley–Terry, and other ranking and paired comparison models (Thurstone, 1927; Bradley and Terry, 1952; David, 1988; Lipovetsky and Conklin, 2003), and on the other hand, discrete choice modeling (DCM) which permits the simultaneous presenting of several items to respondents and estimation of the utility parameters and choice probabilities with a multinomial-logit (MNL) model (McFadden, 1973, 2000; Louviere et al., 2000; McFadden and Richter, 1990; Wedel and Kamakura, 1999; Conklin and Lipovetsky, 1999; Train, 2003; Lipovetsky, 2011; Changpetch and Lin, 2012). Best-Worst Scaling and MaxDiff terms are often used interchangingly but the BWS name is to be preferred – as Louviere and others indicate there is no empirical evidence of humans using a maximum difference measure in a choice process. The choice is actually implemented by respondents answering which of several presented items is the best and which is the worst of them. Each respondent is presented with several (two or three dozen) subsets with a few items in each one, by way of a balanced design where each of the items is shown an approximately equal number of times. As Louviere and his colleagues in Australia Centre for the Study of Choice (CenSoC) have shown (Louviere et al., 2000, 2008, 2013; Street and

* Corresponding author. Tel.: +1 7634174509.
E-mail addresses: stan.lipovetsky@gfk.com (S. Lipovetsky), mike.conklin@gfk.com (M. Conklin).

http://dx.doi.org/10.1016/j.jocm.2014.02.001
1755-5345

Burgess, 2007) the Balanced Incomplete Block Designs (BIBDs) should be preferred as controlling not only for occurrence but also for co-occurrence of the items, which is important as an efficient checking for the Independence of Irrelevant Alternatives (IIA) assumption that the preference between two of alternatives does not depend on the other alternatives.

A normal assumption of BWS is that IIA holds for an individual, although it may not be true (this can be checked by the test of Hausman and McFadden, 1984). BWS can also allow for different best and worst processes, including differences in the utilities (or preferences), the scale (error variances), or both between and within respondents. More complicated models, for instance, corresponding to the branded labeled options with alternative specific attributes, or "availability" designs with a two-stage choice process (Louviere et al., 2000) are also known. An important question is also about availability of alternatives in the hypothesized sequence of the choices taken in each response. For instance, if an item is chosen as the best one, it cannot be used in the estimation of the worst one, and vice versa ("top-down" and other schemes of modeling described in Marley and Louviere, 2005; Flynn et al., 2007; Louviere et al., 2008; Marley et al., 2008). The estimation of utilities in BWS is usually performed using MNL.

The aim of this paper consists of making BWS more clear and open for understanding and interpretation. This paper describes how a close structure of MNL results for utilities and choice probabilities from BWS can be obtained simply from raw response data by analytical formulae of a closed-form. We show that the simple non-iterative arithmetical BWS estimates are close to those obtained in much more complicated approaches, so the analytical solution is very convenient especially for practical calculations on data sets of big base sizes. The modeling used in this paper can be seen as a reasonable first approximation to the true but unknown individual choice processes, and the analytical results based on raw data present an approximation of the choice probabilities. We consider the so-called case 1 of BWS which ranks the items as entire entities, similarly, for instance, to Thursone scaling, while the cases 2 and 3 correspond rather to adaptive and full profile conjoint DCM with comparisons of the items by subsets or by all multiple attributes (Flynn, 2010; Louviere et al., 2013). It is possible to extend the suggested technique to multiple attributes, but it would make the analytical results more complicated so we do not consider them in this current paper. We also do not consider the problem of dealing with inconsistent, tied, and incomplete rankings, rank reversal and circular priority problems in BWS data. Extensive discussion on these inconsistencies has been fulfilled in the literature on binary comparisons (Allison and Christakis, 1994; Brown et al., 2008) and in the Analytical Hierarchy Process (Saaty, 1994; Lipovetsky, 2009).

The paper is constructed as follows. Section 2 presents regular BWS as a special case of a DCM technique, and Section 3 describes BWS in an analytical closed-form solution. Section 4 compares numerical results, and Section 5 summarizes.

2. DCM and BWS

For explicit presentation of the techniques, let us consider an example from a real marketing research project for prioritizing seventeen items. Each of 3062 respondents saw four out of all seventeen items in each of the ten tasks from which the best and worst of the four were chosen. Table 1 presents examples of the items sets shown to two respondents. This way a data matrix with all $3062 \times 10 = 30{,}620$ rows is constructed.

As it is used in DCM applications, each row in Table 1 can be presented in several rows, by the number of the items shown in each task. Table 2 presents such a DCM extended matrix of data for the first respondent and three first sets of

Table 1
Examples of the items shown and the choices made by two respondents.

Resp	Set	Items shown																	Items chosen	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Best	Worst
1	1	0	0	0	0	1	1	0	0	0	1	0	0	0	1	0	0	0	14	6
1	2	0	0	0	0	1	0	1	0	1	0	0	0	0	1	0	0	0	7	5
1	3	1	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	13	9
1	4	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	1	13	17
1	5	1	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	12	1
1	6	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0	1	3	11
1	7	0	1	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	3	2
1	8	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	12	8
1	9	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	1	0	15	11
1	10	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	1	13	8
2	1	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	1	7	11
2	2	0	0	1	0	0	0	0	0	0	1	0	0	1	1	0	0	0	3	10
2	3	1	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	10	6
2	4	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	1	0	5	11
2	5	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	1	12	8
2	6	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	5	17
2	7	0	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0	9	8
2	8	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	1	0	12	6
2	9	0	1	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0	9	2
2	10	1	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	3	1