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On the path independence conditions for discrete-continuous demand

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ABSTRACT

We consider the manner in which the well-established path independence conditions apply to Small and Rosen's (1981) problem of discrete-continuous demand, focussing especially upon the restricted case of discrete choice (probabilistic) demand. We note that the consumer surplus measure promoted by Small and Rosen, which is specific to the probabilistic demand, imposes path independence to price changes *a priori*. We find that path independence to income changes can further be imposed provided a numeraire good is available in the consumption set. We show that, for practical purposes, McFadden's (1981) 'residual income' specification of the conditional indirect utility function offers an appropriate means of representing path independence to price and income changes.

1. Introduction

In common with colleagues applying continuous demand models, economists practised in discrete choice modelling have an interest in the impacts of price and income changes on demand and welfare. The paper by Small and Rosen (1981) (referred to henceforth as 'S&R') has been particularly influential in exploring the interface between continuous demand models—which might be regarded as the convention—and discrete choice models. S&R outline a model of discrete-continuous demand, whereby an individual selects from a set of mutually exclusive alternatives and, conditioned by that choice, consumes a positive quantity of the selected good. Within the context of this model, S&R isolate the consumer surplus change specific to the discrete choice (probabilistic) demand, associated with a change in price, income or some other qualitative attribute of the good in question.

When measuring consumer surplus in any demand context—discrete choice or otherwise—an issue of particular relevance is the welfare impact of income changes, following from a lump sum income supplement/reduction and/or an increase/decrease in real income associated with a price change. As is well established in the literature, the change in Marshallian consumer surplus, which derives from the integration of the Marshallian demand function with respect to the relevant price and income changes, is sensitive to the path of integration (i.e. the sequence of price and income changes). By contrast, the integral of the Hicksian demand function is independent of the path of integration.

S&R's consumer surplus measure is defined in terms of a representative consumer (Gorman, 1953), and conveniently allows the aggregation of discrete choices across repetitions and/or individuals. However, as is widely acknowledged, a limiting property of S&R's measure is that non-linear income effects[1] of price and lump sum income changes are excluded.

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[1] That is to say, income effects which entail a non-linear income expansion path.

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This property straightforwardly ensures path independence (see Morey (1984) for a discussion of path independence more generally), but is somewhat crude, and potentially introduces bias into the resulting measure of surplus. Recognising this limitation, a number of contributors (e.g. Dagsvik and Karlström, 2005; Hau, 1985; Herriges and Kling, 1999; Jara-Díaz and Videla, 1989, 1990; Karlström, 1999; Karlström and Morey, 2001; McFadden, 1995) have explored methods for estimating the Hicksian compensating variation. The attraction of the compensating variation—relative to S&R's measure—is that it elicits a path independent measure of consumer surplus, even when non-linear income effects are present.

Despite this interest in Hicksian surplus measures, the extant literature offers no authoritative commentary on the path independence conditions for discrete choice. The present paper endeavours to fill this gap in the literature. The specific objectives of the paper are:

- To outline the path independence conditions applicable to the discrete-continuous demand in general, and the probabilistic demand (associated with discrete choice) in particular.
- To relate these conditions to the assumptions underpinning the derivation of S&R's consumer surplus measure.
- To draw implications for the practical specification of discrete choice models.

2. Deriving consumer surplus from a model of discrete-continuous demand

This section will introduce notation and, for the benefit of readers unfamiliar with the subject area, briefly summarise the salient features of S&R's model of discrete-continuous demand. Readers already initiated in S&R may wish to proceed directly to Section 3.

2.1. S&R'S model of discrete-continuous demand

Following S&R, consider a maximisation problem wherein the individual consumes non-negative quantities of three goods. Let us assume that goods 1 and 2 are mutually exclusive, whilst the third good—which we refer to as good n—acts as a numeraire. We might think of the latter, more intuitively, as 'all other goods'.

Defining notation: u is direct utility; $\mathbf{x}=(x_1,x_2,x_n)$ is a bundle comprising the quantities of goods 1, 2 and the numeraire good; $\mathbf{p}=(p_1,p_2,1)$ is the associated vector of prices of goods 1, 2 and n (noting that the price of the numeraire good is normalised to one); Y is total income; and y_{1+2} is the income share available to goods 1 and 2 once the numeraire good has been accounted for (i.e. $y_{1+2}=y–x_n$, alluding to the potential for combining good 1 or 2 with good n to form composite goods). We are now equipped to formalise S&R's maximisation problem, as follows:

$$\begin{aligned} &\text{Max} \quad u=u(\mathbf{x}) \\ &\text{s.t.} \quad p_1x_1+p_2x_2=y_{1+2} \\ &\quad x_1x_2=0 \\ &\quad \mathbf{x}\geq 0 \end{aligned} \tag{1}$$

where $y_{1+2}=y–x_n$

An important feature of (1) is the constraint $x_1x_2=0$, which precludes joint consumption of goods 1 and 2. Indeed, S&R conceptualise (1) as a problem of discrete-continuous demand, whereby the individual first chooses between goods 1 and 2 according to which yields the greater utility:

$$u^*(\mathbf{x})=v^*(\mathbf{p},y)=\tilde{v}_k(p_k,y)=\text{Max}\{\tilde{v}_1(p_1,y),\tilde{v}_2(p_2,y)\} \tag{2}$$

where u^* is the maximum direct utility both unconditionally and conditionally given income y, v^* is the maximum indirect utility, $\tilde{v}_k$ is the conditional indirect utility, and k indexes the chosen (i.e. utility maximising) good, i.e. $k=1$ if $\tilde{v}_1\geq\tilde{v}_2$, or $k=2$ otherwise. Having chosen between goods 1 and 2, the individual selects a positive quantity of the chosen good, as well as a non-negative quantity of the numeraire good. If income is devoted entirely to goods 1 and 2, then $y=y_{1+2}$ and consumption of the numeraire good will be zero.

As the annex to the present paper shows, if we solve (1) for the uncompensated demands for goods 1 and 2 then, unlike more conventional continuous demand models, Roy's identity derives the demands for goods 1 and 2 *conditional* upon the discrete choice between goods 1 and 2:

$$-\frac{\partial v^*(\mathbf{p},y)/\partial p_1}{\partial v^*(\mathbf{p},y)/\partial y}=\begin{cases} -\frac{\partial\tilde{v}_1(p_1,y)/\partial p_1}{\partial\tilde{v}_1(p_1,y)/\partial y}=\tilde{x}_1 & \text{if } k=1 \\ -\frac{\partial\tilde{v}_2(p_2,y)/\partial p_1}{\partial\tilde{v}_2(p_2,y)/\partial y}=0 & \text{if } k=2 \end{cases} \tag{3}$$

$$-\frac{\partial v^*(\mathbf{p},y)/\partial p_2}{\partial v^*(\mathbf{p},y)/\partial y}=\begin{cases} -\frac{\partial\tilde{v}_1(p_1,y)/\partial p_2}{\partial\tilde{v}_1(p_1,y)/\partial y}=0 & \text{if } k=1 \\ -\frac{\partial\tilde{v}_2(p_2,y)/\partial p_2}{\partial\tilde{v}_2(p_2,y)/\partial y}=\tilde{x}_2 & \text{if } k=2 \end{cases}$$

where $\tilde{x}_j$ is uncompensated demand conditional upon the choice of good $j=1,2$.