Journal of Corporate Finance 31 (2015) 91–115

Contents lists available at ScienceDirect

Journal of Corporate Finance

journal homepage: www.elsevier.com/locate/jcorpfin

Valuing convertible bonds and the option to exchange bonds for stock

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ARTICLE INFO

Article history:
Received 10 November 2014
Received in revised form 20 November 2014
Accepted 26 December 2014
Available online 6 January 2015

JEL classification:
C29
G12
G13
G30
G32

Keywords:
Valuation
Convertible bond
Exchange option
Stopping time
Empirical validation

ABSTRACT

The value of a conventional convertible bond is the value of a straight bond plus the value of the option to exchange it for a specified number of shares of common stock. First, I develop a closed-form contingent-claims convertible bond valuation model that quantifies the value of the exchange option when the short-term riskless rate, the firm's credit spread, and its share price are stochastic. I model the firm's decision to force early conversion as a stopping time problem in which the firm forces conversion as soon as the conversion value reaches the forced conversion barrier. I empirically validate the model by comparing model and market prices for a sample of 148 corporate convertible bonds issued between 2006 and 2010. The average median and mean pricing errors are −0.18% and 0.21%, respectively, which are within the average bid–ask spread for convertible bonds during the sample period. I use the model to quantify the disruptive impact that the prohibition on short selling during the recent financial crisis had on convertible bond prices.

1. Introduction

Convertible bond valuation is not amenable to an exact closed-form solution because of the security's complex optionality. A convertible bond gives the holder an American option to convert the bond into common stock by exchanging it for a specified number of common shares at any time prior to the bond's redemption. Often, the firm has an American call option, which it can use to force conversion before the bondholders voluntarily convert, if the conversion option is in-the-money, and the bondholders may have one or more European put options, which they can use to force premature redemption. The interaction of these options with the firm's default option requires a contingent claims valuation model to capture fully a convertible bond's complex optionality. However, an accurate approximation can be achieved if the exchange option and these option interactions can be modeled appropriately.

This paper makes four contributions to the corporate finance literature. First, it demonstrates that modeling a conventional convertible bond as a straight bond coupled with an option to exchange the bond for a specified number of common shares can lead to a closed-form expression that provides an accurate approximation to the convertible bond's value. It builds on Ingersoll's (1977a) insight that a convertible bond can be viewed as a combination of a straight bond and an option to exchange it for the underlying common stock and draws on Margrabe's (1978) insight concerning the valuation of an option to exchange one asset for another. Second, I develop a procedure for estimating the expected forced conversion date of a callable convertible bond by treating the firm's

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http://dx.doi.org/10.1016/j.jcorpfin.2014.12.012

decision to force early conversion as a stopping time problem in which the firm forces conversion as soon as the conversion value reaches a forced conversion barrier. Third, I empirically test the model on a sample of 148 convertible bonds issued between January 2, 2006 and December 31, 2010 based on TRACE prices for the January 31, 2006 through January 31, 2014 time period. I find that the overall average median and mean pricing errors are −0.18% and 0.21%, respectively, which are within the average bid–ask spread for the convertible bond sample. Fourth, I use the model to quantify the disruptive impact that the SEC's short selling restrictions had on convertible bond prices during the recent financial crisis.

A corporate bond is both an interest-rate derivative and a credit derivative. I model the evolution of the riskless rate and the credit spread as correlated mean-reverting diffusion processes. The credit spread process incorporates default risk within a reduced-form model. I model the investors' option to exchange the straight bond component for the conversion shares and develop a closed-form convertible bond valuation model. The strike price is the market value of the straight bond component, which varies with interest rates and the bond's credit risk. I extend Margrabe's (1978) model to obtain an explicit expression for the value of the option to exchange the straight bond for the conversion shares. I extend the model to callable convertibles by incorporating the firm's option to force early conversion within a stopping time framework. I further explain how to apply the model to value putable convertibles.

The exchange option convertible bond pricing model is simpler to use than the more mathematically sophisticated partial differential equation (PDE) models. A simple mathematical model can have practical utility. For example, the Black–Scholes–Merton model is still the most widely used equity option pricing model despite its simplifying assumptions that stock price volatility and the riskless rate are both constant during the life of the option. Option market participants have found ways to adapt the model, for example, by fitting the Implied Volatility Function to take into account the volatility smile when estimating the volatility parameter (Hull and Suo, 2002). In addition, a model that fits the bond with exchange option intuition and permits the separate valuation of the exchange option facilitates a more intuitive valuation of convertible bonds. I develop an explicit expression for the value of the exchange option. Finally, closed-form models allow comparative statics to be calculated more easily than PDE or lattice models. Appendix C furnishes a full set of comparative statics for my model.

The rest of the paper is organized as follows. Section 2 relates the paper to the convertible securities valuation literature. Section 3 develops my model. Section 4 extends the model to value callable and putable convertible bonds within a stopping time framework for estimating the forced conversion date for callable convertibles. Section 5 empirically tests the model on a sample of 148 convertible bonds issued between 2006 and 2010 and quantifies the impact of the SEC's short selling restrictions during the financial crisis. Section 6 concludes.

2. Convertible securities valuation literature

The convertible securities valuation literature is firmly rooted in contingent claims modeling, which originated with the seminal papers by Brennan and Schwartz (1977) and Ingersoll (1977a). In both papers, the value of the firm's assets follows geometric Brownian motion; the firm's equity and convertible securities are contingent claims on the value of its assets; and default is modeled within Merton's (1974) single-factor structural framework. The papers provide similar pricing insights but whereas Ingersoll (1977a) derives analytic valuation formulas for several special cases, Brennan and Schwartz (1977) develop a general algorithm for valuing a callable convertible bond. This general framework accommodates discrete coupon payments and dividends, conversion terms that change contractually during the bond's life, and call protection that restricts the issuer's ability to exercise the call option. Importantly, both papers prescribe that in a frictionless market when there are no restrictions on call option exercise, the firm's optimal call strategy for minimizing the convertible's value at each instant in time is to force conversion as soon as the bond's conversion value first reaches the effective call price (the stated call price plus accrued interest).

Brennan and Schwartz (1980) extend Brennan and Schwartz (1977) by assuming that the short-term riskless rate follows a mean-reverting lognormal stochastic process. In both models, the firm might default on the convertible bond at maturity, in which case bondholders receive a fixed fraction of the face value. Brennan and Schwartz (1980) demonstrate that under reasonable assumptions about the interest-rate process, assuming a non-stochastic riskless rate would introduce errors of less than 4%. McConnell and Schwartz (1986) extend the Brennan and Schwartz (1980) model to value zero-coupon convertible bonds that provide for a series of embedded firm call options and investor put options.

A rich convertible security pricing literature has evolved as researchers have sought to develop and empirically test more tractable contingent claims models that capture their complex optionality. Nyborg (1996) compares PDE models and the single-factor lattice model. In practice, the single-factor binomial lattice model is one of the most widely used convertible security valuation models (Bhattacharya, 2012; Hull, 2012). These models take two important shortcuts. They assume a constant riskless rate (thus ignoring interest rate volatility) and a constant credit spread (thus ignoring credit spread volatility) to capture the default risk and model the convertible bond as a contingent claim on a single factor, the firm's stock price. For example, Tsiveriotis and Fernandes (1998) describe a reduced-form lattice model that decomposes convertible bond value into two components. One applies when the conversion feature is not exercised and the security winds up as debt. Payments are discounted at the riskless interest rate plus a credit spread. The other applies when the conversion feature is exercised and the bond winds up as common equity. Payments are discounted at the riskless interest rate. Ammann et al. (2003) test the model on a sample of 21 French convertible bonds and find that the model prices are on average more than 3% higher than market prices and that the overpricing is most severe for out-of-the-money convertibles.

The lattice approach can handle more than one factor but simplifying assumptions are required to achieve tractability. Takahashi et al. (2001) develop a reduced-form PDE model incorporating Duffie and Singleton's (1999, 2003) default risk model and compare the pricing accuracy of their default-boundary trinomial lattice model, and the models in Cheung and Nelken (1994), Goldman Sachs (1994), and Tsiveriotis and Fernandes (1998) for four non-callable and non-putable Japanese convertibles. Their model generally performs best