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Computer aided geometric design of strip using developable Bézier patches

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ABSTRACT

Developable strip is commonly used in product design due to its ease of manufacture. This paper proposes an algorithm for geometric design of strip using developable Bézier patches. It computes an aggregate of triangular and quadrilateral patches interpolate two given space curves defining a strip. The computation process selects optimal solutions in terms of surface assessment criteria specified by the user. Each patch is then degree-elevated to gain extra degrees of freedom, which produce G1 across the patch boundaries by modifying the control points while preserving the surface developability. Test examples with different design parameters illustrate and validate the feasibility of the proposed algorithm. In comparison with previous studies, this work allows strip design with freeform developable patches, generates better results in the surface assessment, and provides more flexible control on the design shape. It serves as a simple but effective approach for computer aided geometric design of developable strip.

1. Introduction

Developable surfaces are a subset of ruled surfaces which can be unfolded (or developed) into a plane without tearing or stretching during the process. This property, known as the developability, eases manufacture of 3D objects. Hence, developable shapes are widely used in industries such as sheet metal forming [1], ship building [2,3], windshield design, and fabrication of apparels including shoes and clothing [4,5]. Parts are first modeled with developable strips in space. They are then flattened into a planner pattern. The manufacturing process starts with cutting a material according to the pattern. Unrolling the cut material simply resumes its original 3D shape. A final step is often applied to assemble different pieces by welding or sewing in order to form the final product.

There have been two different approaches proposed for CAGD of a single developable surface. The first approach represents a surface as a tensor product of degree $(1, n)$ with non-linear constraints imposed by the developability. The user can thus control the shape in a very limited manner, e.g. some but not all of the control points. The remaining parameters must be solved from the constrained system [6–11]. Instead, one can treat a developable surface as an envelope of one parameter set of tangent planes. The surface becomes a curve in dual projective space [12]. Design methods were proposed for Bézier and B-spline surfaces based on the duality theory [13,14]. However, they may be lacking of practicality in CAGD applications.

Many engineering products consist of double-curved surfaces, which are not perfectly developable. It is thus necessary to allow certain deviations in the developability. Several studies have developed CAGD methods that approximate 3D shape using developable surfaces. Some concerned with interpolation and approximation algorithms based on the dual approach [15–19]. Leopoldseder and Pottmann [16] modeled a given developable surface by surfaces of revolution. Each pair of consecutive rulings and tangent planes that approximates the given surface is interpolated by smoothly linked circular cones. Their later work allowed a point cloud as the input for applications in reverse engineering [17,18]. On the other hand, several literatures [20,21,22–24] were focused on increasing the developability of a strip in the tessellation representation. Wang et al. [20] proposed a function optimization method for increasing the developability of a trimmed NURBS surface by adjusting the positions and weights of the surface control points. Tang and Wang [22] introduced a modeling algorithm that interpolates a strip defined by two given

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space curves with an aggregate of triangles. The interpolation task was formulated as a variant of boundary triangulations and thus transformed into the shortest-path problem [23]. Their later work [24] optimized the result based on various objective functions considering different CAD/CAM applications.

Strip design using developable surfaces possesses a wide range of industrial applications. It is commonly used in design and manufacture of sheet metal parts and apparel. Developable strip design has recently found novel applications such as generation of tool path in five-axis flank milling [25], simulation of robot motions [26], and fabrication of sculptures in art [27]. However, the past studies fail to provide geometric design methods that balance the modeling capability and usability in practice. This paper introduces a greedy algorithm for CAGD of strip with freeform developable patches. It calculates consecutive quadratic Bézier patches in the conical form, consisting of triangular and quadrilateral patches, that interpolates two given boundary curves in space. Simple heuristics are applied to select one optimal solution in terms of surface evaluation criteria among the feasible patches starting with a ruling defined by two sampling points from the curves. The patches generated connect with only positional continuity. The next step is to perform degree elevation on each patch. The resultant cubic patch provides extra degrees of freedom in the strip design. G1 continuity is thus produced across the degree-elevated patches by adjusting their control polygons while maintaining the surface developability. Test strips defined by highly convoluted curves demonstrate the effectiveness of the proposed method. The influence of various design parameters on the strip shape is discussed. In comparison with previous research that employed triangles in the strip design, this work offers better surface developability, simpler solution for quick implementation, and more design handles for the shape control. It serves as a simple but practical method for CAGD of developable strip.

2. Preliminaries

2.1. Developable Bézier patch

Given two curves $\mathbf{P}(u)$ and $\mathbf{Q}(u)$ in 3D space, a ruled surface is constructed by linking each pair of corresponding curve points (with equal u) with a line segment $\mathbf{PQ}$, referred to as a ruling. The surface $\mathbf{R}$ is described as

$$\mathbf{R}(t,u) = (1-t)\,\mathbf{P}(u) + t\,\mathbf{Q}(u), \quad (t,u) \in [0,1] \times [0,1] \tag{1}$$

where t is the parameter along the ruling. Generally the tangent lines to the curves $\mathbf{P}(u)$ and $\mathbf{Q}(u)$ at any given point do not lie in the same plane. If these tangent lines and the corresponding ruling remain coplanar, then the surface becomes developable, which can be represented in terms of the triple scalar product of the two tangent vectors and the ruling vector $\mathbf{P}(u) - \mathbf{Q}(u)$ [9]:

$$\dot{\mathbf{P}}(u) \times \dot{\mathbf{Q}}(u) \cdot [\mathbf{P}(u) - \mathbf{Q}(u)] = 0 \tag{2}$$

Substituting the freeform representation of both curves into Eq. (2) leads to a complex system of equations that must be imposed on the control points to ensure the surface developability.

Imposing proper geometric restrictions on a patch can sometimes simplify the developability constraints and the solution process of the constrained control points. Several previous studies [6,7,10] applied this technique to make the surface design solvable. The resultant patches are special cases of the most general developable patch. Certainly these pre-defined limitations consume some degrees of freedom in the patch design, and thus reduce the modeling capability of the surface. For a developable Bézier patch, when the extensions of all the trapezoids in the Bézier

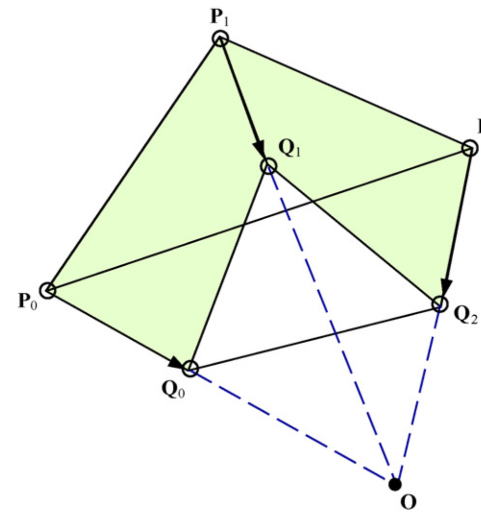

Fig. 1. A quadratic Bézier patch in generalized conical form.

control polyhedron intersect at a point $\mathbf{O}$ (see Fig. 1) and vectors $\mathbf{c}_0$, $\mathbf{c}_1$, and $\mathbf{c}_2$ in the patch must satisfy [9]:

$$\frac{\mathbf{c}_0}{\mathbf{P}_0\mathbf{O}} = \frac{\mathbf{c}_1}{\mathbf{P}_1\mathbf{O}} = \frac{\mathbf{c}_2}{\mathbf{P}_2\mathbf{O}} = f \tag{3}$$

where f and $\mathbf{O}$ are referred to as the scaling factor and the projection point, respectively. The patch becomes developable and refers to as the generalized conical form [9]. Eq. (3) indicates that one boundary curve is simply a scaled copy of the other curve. This conclusion also reveals that any control point pairs must remain coplanar. Any Bézier ruled patch with one boundary reduced into a single point is a triangular developable patch [28]. That is, any surface constructed by linking from a projection point to a Bézier curve becomes developable. Despite its limited modeling capability, the conical model provides simple but useful design methods in many applications of developable patches.

3. Modeling with developable Bézier patch

A strip is defined by two boundary curves. They are defined by piecewise parametric curves interpolating a set of sample points. The strip design in this work is to find a series of developable surfaces that interpolate the two curves. Assume they are denoted as $\mathbf{P}(u)$ and $\mathbf{Q}(u)$, the first step is to take a set of points $P = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n\}$ and $Q = \{\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_m\}$. To interpolate P and Q with maximal developability is a variation optimization problem [24]. This paper proposes a different approach which does not need to solve such a complex problem. The idea is to use developable patches to interpolate P and Q so that the developability is automatically preserved. At any ruling $\mathbf{p}_i\mathbf{q}_j$, we can choose two different elements to start with: a triangular or a quadrilateral developable patch. The following algorithms describe how to calculate the control points in each case.

3.1. Generation of a triangular patch

As described above, a triangular developable Bézier patch is defined with a projection point and a Bézier boundary curve. We choose the projection point and the end control points of the curve from the point sets P and Q. A curve constructed in this way would be of close proximity to the boundaries. As shown in Fig. 2, suppose $\mathbf{p}_i$ and $\mathbf{p}_{i+1}$ are two consecutive points in the set P, with the tangent vectors to the original curve denoted as $\mathbf{t}_i$ and $\mathbf{t}_{i+1}$. They serve as the