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Robust econometric inference with mixed integrated and mildly explosive regressors

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ABSTRACT

This paper explores in several prototypical models a convenient inference procedure for nonstationary variable regression that enables robust chi-square testing for a wide class of persistent and endogenous regressors. The approach uses the mechanism of self-generated instruments called IVX instrumentation developed by Magdalinos and Phillips (2009b). We first show that these methods remain valid for regressors with local unit roots in the explosive direction and mildly explosive roots, where the roots are further from unity in the explosive direction than $O\left(n^{-1}\right)$. It is also shown that Wald testing procedures remain robust for multivariate regressors with certain forms of mixed degrees of persistence. These robustifications are useful in econometric inference, for example, when there are periods of mildly explosive trends in some or all of time series employed in the analysis but the exact knowledge on the regressor persistence is unavailable. Some aspects of the choice of the IVX instruments are investigated and practical guidance is provided but the issue of optimal IVX instrument choice remains unresolved. The methods are straightforward to apply in practical work such as predictive regression applications in finance.

1. Introduction

Many economic and financial time series exhibit characteristics that include temporary periods of explosive behavior. For macroeconomic series Stock (1991, Table 2) showed that 90% confidence intervals for the autoregressive (AR) roots of the Nelson–Plosser dataset contain explosive parameter regions in all but one series (the unemployment rate). For financial series Campbell and Yogo (2006, Table 4) found that 95% confidence intervals for the AR coefficient of the S&P 500 dividend–price ratio and other series over long historical periods do not rule out explosive roots. In addition to these empirical findings, periodically occurring booms and episodes of financial exuberance support at least temporary explosive trends in economic and financial data. The idea that there are subperiods of explosive roots in economic and financial series is formally analyzed and empirically confirmed in several recent papers (Phillips et al., 2011, 2015a,b; Phillips and Yu, 2011; Homm and Breitung, 2012).

There has been growing interest in predictive regressions of the type used in the study by Campbell and Yogo. When the regressors in such models display some degree of persistence, inference procedures need to be robust to the value of the persistence parameter to ensure validity even asymptotically. The robustness requirements become more demanding in cases where there are many regressors with possibly different degrees of persistence. To address some of these complexities in inference in regressions with persistence regressors, Magdalinos and Phillips (2009b, hereafter MP) recently introduced a novel IV procedure (called IVX regression) that provides robust chi-square inference in a much wider vicinity of unity than existing studies that have typically only considered the near integrated (local to unity) single regressor case. In particular, MP showed that self-normalized IVX test statistics have an asymptotically pivotal chi-square distribution for multivariate regressors that may be integrated, near integrated, or mildly integrated and which thereby fall within a vector autoregressive framework (Lütkepohl, 2005) while allowing for

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more general time series inputs than martingale differences. The tests have been successfully used in applied work on predictive regressions (Kostakis et al., 2014; Gonzalo and Pitarakis, 2012). IVX methods have been also studied in long-horizon regression applications (Phillips and Lee, 2013) and in quantile regression (Lee, 2016) contexts.

The present paper is an exploratory study in prototypical cases that show how the IVX methodology extends to a wider range of potential regressors that includes local unit root (LUR) explosive and mildly explosive roots, thereby covering periods of exuberance in economic and financial data. The limit theory involves some novel developments in the mildly explosive case, wherein the latent IVX instrument which depends on the true values of the localizing coefficients may no longer dominate the asymptotics. The chi-square limit theory of the same self-normalized test statistics of MP is shown to continue to be valid in this wider setting. We also confirm that the limit theory is robust under mixed degrees of persistence, allowing for the simultaneous presence of local to unity (or mildly integrated) roots and mildly explosive roots. These extensions show that IVX regression provides a framework of test procedures that can cover a large class of persistent regressors whose individual characteristics may differ from each other. While the results in the present paper show validity in a prototypical system, the wider practical implication is that empirical researchers may use this framework without having to be specific about the particular properties of individual regressors. Further research that is presently underway aims to provide a unified treatment that justifies the use of this approach for a comparably wide class of regressors in a general multivariate setting.

The paper is organized as follows. Section 2 develops the limit theory for IVX regression under locally and mildly explosive roots and demonstrates robustness. Section 3 extends these robustness results to cases where the regressors have mixed degrees of persistence or explosive behavior. Section 4 discusses issues associated with the choice of the IVX tuning parameter and provides simulation results. Section 5 concludes. Some technical derivations, supporting lemmas, and proofs of the main results in the paper are contained in the Appendix. An online supplement (Phillips and Lee, 2015b) provides an empirical illustration as well as further lemmas and proofs that are useful in establishing the results of the paper.

2. IVX regression with explosive roots

In what follows, we use the spectral norm $\|M\| = \max_i \{\lambda_i^{1/2} : \lambda_i = \text{an eigenvalue of } M'M\}$. Other norms, such as the L_1 and L_2 norms, are specified in what follows as needed using the notation $\|\cdot\|_{L_i}$ $(i = 1, 2)$.

2.1. Framework

We use the framework of MP for the following system:

$$\begin{aligned} y_t &= Ax_t + u_{0t}, \\ x_t &= R_n x_{t-1} + u_{xt}, \\ R_n &= I_K + \frac{C}{n^\alpha}, \quad \text{for some } \alpha > 0, \end{aligned} \tag{2.1}$$

where A is an $m \times K$ coefficient matrix and $C = diag\,(c_1, c_2, \ldots, c_K)$ represents the localizing coefficients in the multivariate regressors. Use of a diagonal matrix C in (2.1) has become standard practice in the literature because it helps to distinguish individual components of x_t in terms of their integration properties.[1]

The IVX approach of MP allowed the regressors x_t to be (I1) integrated ($C = 0$), (I2) near integrated ($C < 0, \alpha = 1$) and (I3) mildly integrated ($C < 0, \alpha \in (0, 1)$), and developed an inference procedure that is robust to the precise degree of integration.[2] We will show these results are robust under the same framework but with (I4) local to unity on the explosive side of unity ($C > 0, \alpha = 1$) and (I5) mildly explosive roots ($C > 0, \alpha \in (0, 1)$), as well as possibly mixed versions (I6) of these roots.

As in MP or Kostakis et al. (2014), the initial conditions for the driver regressor x_t in (I1)–(I2) and (I3) could be $x_0 = o_p\left(n^{1/2}\right)$ or $x_0 = o_p\left(n^{\alpha/2}\right)$, respectively. Indeed, $x_0 = o_p\left(n^{\alpha/2}\right)$ also applies in the (I4) case. For (I5), in view of Lemmas 2.1 and 2.2, we could impose $x_0 = o_p\left(n^{\alpha/2}\left\|R_n\right\|^n\right) = o_p\left(n^{\alpha/2} e^{\|C\| n^{1-\alpha}}\right)$. Other possibilities, where the initial conditions have larger orders may also be analyzed and, as in Phillips and Magdalinos (2009) such cases typically impact the asymptotic theory.

The system (2.1) is prototypical and may readily be extended to cases that suit particular empirical applications. For example, intercepts and deterministic regressors are commonly present in practice in predictive systems like (2.1). Also, the generating mechanism for the predictors x_t may also involve deterministic components or triangular system versions of such components that upon standardization lead to functional (or other) stochastic process limits with comparable deterministic components (e.g. Phillips et al. (2014)). Theory and implementation of the methods discussed here under some extensions of this type are considered later in the paper.

For the time series structure of the innovations that appear in (2.1) it is convenient to maintain one of the following two assumptions, which allow for weak dependence and conditional heterogeneity.

Assumption 2.1 (*LP*).

$$\begin{aligned} &u_t := \begin{bmatrix} u_{0t} \\ u_{xt} \end{bmatrix} = \sum_{j=0}^{\infty} F_j \varepsilon_{t-j}, \quad \varepsilon_t \sim iid\,(0, \Sigma), \\ &\quad \Sigma > 0,\ E\left\|\varepsilon_1\right\|^4 < \infty, \\ &F_0 = I_{m+K}, \sum_{j=0}^{\infty} j\left\|F_j\right\| < \infty, \qquad F(z) = \sum_{j=0}^{\infty} F_j z^j \quad \text{and} \\ &F(1) = \sum_{j=0}^{\infty} F_j > 0. \end{aligned} \tag{2.2}$$

Assumption 2.2 (*NLP*).

$$u_t = g(\xi_t) \tag{2.3}$$

where the process $\xi_t = (\ldots, \varepsilon_{t-1}, \varepsilon_t)$ consists of time series innovations $\varepsilon_t \sim iid\,(0, \Sigma)$ as given Assumption LP above, and where $g(\cdot)$ is a measurable function such that u_t is well-defined. Let $\left\{\varepsilon_i'\right\}_{i \in \mathbb{Z}}$ be an iid copy of $\{\varepsilon_i\}_{i \in \mathbb{Z}}$. The following geometric moment contraction (GMC) condition holds, for all $t \in N$ with $M > 0$ and $r \in (0, 1)$,

$$E\left[\left\|u_t - u_t^*\right\|^q\right] \le Mr^t, \quad \text{with } q \ge 4$$

where $u_t^* = g(\ldots, \varepsilon_{-1}', \varepsilon_0', \varepsilon_1, \ldots, \varepsilon_{t-1}, \varepsilon_t)$.

[1] The general case of matrix C was considered in the original work on near integrated vector processes (Phillips, 1988) with several examples that illustrate the impact of off diagonal elements on the limit theory. These typically introduce additional components in the limit that take the form of smoothed diffusions.

[2] Some other cases of neglected nonlinearities (such as structural breaks) and linkages to unit root or explosive behavior may be included in the set up. For instance, Diebold and Inoue (2001) have shown that stochastic regime switching is easily confused empirically with long memory; and certain nonlinear functions of $I(1)$ processes are well-known to behave like stationary long memory processes (Miller and Park, 2010; Kasparis et al., 2014).