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Structural estimation of pairwise stable networks with nonnegative externality[☆]

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ABSTRACT

This paper develops a framework to structurally estimate pairwise stable networks with nonnegative externality. We characterize pairwise stable equilibria as a fixed point of a certain mapping and show that the set of pairwise stable equilibria with nonnegative externality is a complete lattice. We extend the characterization to an econometric framework for structural estimation based on the moment inequality model. We apply our methodology to friendship networks of students in the United States, using data from Add-Health. We find that the preference toward racial homophily is overestimated if we do not control for the preference toward clustering.

1. Introduction

Since the seminal paper of Jackson and Wolinsky (1996), the theory of the economics of networks has been extensively studied and applied to numerous fields.[1] Many theories and applications that consider network formation, including Jackson and Wolinsky (1996), use the equilibrium concept known as "pairwise stability". This equilibrium concept basically requires that no pairs have an incentive to deviate from the current network structure. The concept is appealing because of its simplicity and intuition and has been applied to many fields, such as collaborative research and development networks (Goyal and Joshi, 2003), free-trade networks (Goyal and Joshi, 2006a), and risk-sharing networks (Kranton and Bramoullé, 2007).

However, few papers have conducted the structural estimation of network formation models using this equilibrium concept. One of the reasons for this is a lack of characterization. Neither the existence nor the uniqueness of the equilibrium is ensured in general. Although the issue of existence in a generic case is examined by Jackson and Watts (2002) where they characterize sufficient conditions for the existence of the equilibrium based on the concept of "improving paths", their conditions are difficult to check, so their approach is not directly applicable upon estimation. The lack of an econometric framework for pairwise stable network formation has been an obstacle to the development of the field.

This is not merely a theoretical concern. For example, in the context of friendship network formation, economists and sociologists have extensively studied the degree of "racial homophily" in a friendship network, i.e. the degree to which people of the same race are likely to be friends with each other (Lazarsfeld and Merton, 1954). This parameter is of particular importance when we

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1 See Jackson (2008) for a survey.

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consider racial integration policies. For example, Moody (2001) conducts a regression of whether a pair of people are friends with each other on a dummy variable that indicates whether the pair are of the same race. He concludes that even racially integrated schools may still exhibit racial segregation in friendships if the degree of racial homophily is high. However, it is also pointed out in the literature that there exists a tendency that a pair with a mutual friend are more likely to be friends, i.e. clustering (Simmel, 1908). Then, omitting the term of existence of mutual friend in the regression as in Moody (2001) is likely to overestimate the coefficient on the same-race dummy.[2]

To deal with the stated difficulty associated with the lack of existence and characterization of equilibria, we take an approach that restricts the form of utility functions. By assuming that there is no negative externality, i.e. the incentive to form a link weakly increases as the number of links increases, we show that the set of equilibria is a nonempty complete lattice. The concept of nonnegative externalities is not new; it is the same as "strategic complementarity and convexity in own links" studied in Hellmann (2013). Hellmann (2013) showed the existence of an equilibrium using the approach of Jackson and Watts (2002). However, we adopt a different approach for the proof. We characterize the equilibrium as a fixed point of a certain mapping and apply Tarski's fixed-point theorem (Tarski, 1955). The value added of this approach is that we can show that the set of equilibria is a complete lattice, in addition to existence. This implies that there is a unique maximum equilibrium and minimum equilibrium under a suitably defined partial order, and all equilibria lie in between them. This characterization is essentially an application of Topkis (1979), Milgrom and Roberts (1990) and Vives (1990), although in the context of network formation, no paper thus far has proposed a characterization of the equilibrium with this approach.

Using the equilibrium characterization above, we provide a framework for structural estimation. The basic approach is to use moments that maintain the partial ordering. Then, the moments of the observed network exist in between that of the minimum and the maximum equilibria. In this way, we can construct a moment-inequality model without assuming any equilibrium selection rule other than that of pairwise stability. This approach of utilizing Tarski's fixed-point theorem for structural estimation is conducted in Jia (2008), Nishida (2015), Uetake and Watanabe (2013b) and Uetake and Watanabe (2013a) in different contexts. We review the relationship with these papers in Section 2.

We then apply our framework to friendship networks from the National Longitudinal Study of Adolescent Health (Add Health). The dataset contains data on U.S. students in grades 7–12 from a nationally representative sample of roughly 130 private and public schools during the years 1994–1995.

Our empirical objective is to obtain reliable estimates of the preference toward racial homophily after controlling for the preference toward clustering. We find that omitting the variable of clustering overestimates the degree of racial homophily (in a sense that larger values of the parameters are more likely to be rejected and smaller values of the parameters are more likely to be accepted). This finding has an important implication for racial integration policy, such as class assignment in schools.

The rest of the paper is organized as follows. Section 2 summarizes the related literature. Section 3 describes the model and the theoretical characterization of the equilibria. Section 4 introduces an econometric model and explains how we construct moment inequalities. Section 5 provides the results of Monte-Carlo simulation. Section 6 explains the application of our method to the student friendship networks. Section 7 presents our conclusion.

2. Related literature

2.1. Structural estimation of network formation models

There are several recent papers that attempt to structurally estimate network formation models with different approaches. One strand of the literature takes the approach of exploring an entirely different equilibrium concept than that here. Christakis et al. (2010) models network formation as a sequential process where in each period a single, randomly-selected pair of agents has the opportunity to form a link, and Mele (2016) considers a stationary distribution of such a process. Chandrasekhar and Jackson (2014) investigates the approach based on random graph models, which have been developed outside economics (i.e. Kolaczyk (2009)).

There are also several recent papers that employ the same equilibrium concept of pairwise stability as this paper, such as Sheng (2014), de Paula et al. (2015), and Leung (2016). Sheng (2014) conducts moment inequality inference based on the concept of "subnetworks". de Paula et al. (2015) consider a new model set-up where similarly defined subnetworks carry over enough information to characterize the identified set of the parameters, and provide a method to compute the identified set. Leung (2016) takes a completely different approach and develops a weak law of large numbers for sparse networks. In this paper, we impose different assumptions and take an entirely different approach for inference.

2.2. Structural estimation of supermodular games

The method developed in this paper is one application of the growing literature on the structural estimation of supermodular games. On the theoretical side, the characterization of the equilibrium in this paper is essentially an application of Topkis (1979), Milgrom and Roberts (1990) and Vives (1990), although in the context of network formation, no paper we know of has proposed a characterization of the equilibrium with an approach similar to ours.

Empirically, many papers have utilized the characterization of equilibria of supermodular games for structural estimation. Most notably, Jia (2008) considers the entry game between Walmart and Kmart. Nishida (2015) extends Jia (2008)'s result to the situation where each brand can choose multiple branches in the Japanese convenience-store industry. Both of these papers assume extremal equilibria of the lattice as an equilibrium selection rule, and they obtain two different estimates corresponding to each equilibrium. On the other hand, Uetake and Watanabe (2013b) illustrates how the lattice equilibrium structure of a noncooperative supermodular game can be translated to moment inequalities.

The methodology in this paper of utilizing Tarski's fixed-point theorem for structural estimation should be viewed as an extension of Jia (2008), Nishida (2015) and Uetake and Watanabe (2013b). How we translate lattice equilibrium structure to moment inequalities closely follows Uetake and Watanabe (2013b), though we consider a more general construction of moment inequalities.[3]

2.3. Structural estimation of two-sided matching models

There have been substantial developments in the literature of structural estimation of two-sided matching. Unlike network

[2] The intuition behind this bias is stated in more detail in Section 4.4.

[3] Section 4 describes this point in more detail.