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Social interactions under incomplete information with heterogeneous expectations[☆]

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ABSTRACT

We analyze social interactions where the conditional expectations about group members' behaviors are heterogeneous with individual features as well as asymmetric private information, under the framework of a simultaneous move game with incomplete information. A functional contraction mapping is used to establish the existence of a unique Bayesian Nash equilibrium. The method of nested fixed point maximum likelihood estimation performs well for both linear and binary choice models. If heterogeneity is assumed away, estimates will be biased. For the 2011 National Youth Tobacco Survey data, significant peer effects on juvenile tobacco use are found.

1. Introduction

It is natural to believe that an agent may not know features of all other members in her social group. For example, a college student might know the SAT scores of her close friends in a class but not those of others. Because of different information, two students with the same characteristics may have different predictions on academic performance of a third student. Hence, heterogeneity in private information may influence an individual's actions through social interactions. However, as far as we know, estimation of social interaction models with this type of heterogeneity has not been analyzed in the existing literature. When the self-known personal characteristics are modeled as independent shocks across agents and are assumed to be uncorrelated with the publicly known individual features, an agent's expectation on the behavior of other group members will only depend on the public information. That is why any two agents form the same expectation on the behavior of a third individual, in models by Manski (1993), Brock and Durlauf (2001), Lee et al. (2014), and Xu (2012). Blume et al. (2015) build a linear social interaction model on the basis of a simultaneous move game under incomplete information with a general form of the joint distribution of the publicly known characteristics and the self-known shocks. In that case, it is possible for two agents to make different predictions. In this paper, we keep the conventional assumption that the self-known shocks are independent of each other and individual characteristics are exogenous. Instead, there may be private information on the exogenous characteristics. We build a model based on a simultaneous move game under incomplete information and investigate the existence and uniqueness of equilibrium, computation, model identification, and estimation.

By imposing a structure on expected utilities of individuals in a game, we are able to build a behavioral model incorporating different types of actions, continuous and discrete, under various information structures. In this model, an agent's action depends on her expectations on the behaviors of her linked agents, based on her information about group features, social relationships, and

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some specific personal characteristics. There is a correspondence between a Bayesian Nash Equilibrium (BNE) and the equilibrium conditional expectations on members' behaviors. The equilibrium expectation function shows two types of heterogeneity. On one hand, the expectation on the behaviors of two different agents are generally different, for they have different traits. On the other, if the private information of two agents differ, their expectations on a third person may not be the same. As the equilibrium conditional expectation is a vector-valued function of privately known individual characteristics, we embed conditional expectations in a function space and use functional contraction mapping to derive a sufficient condition for the existence of a unique BNE. It is shown that for some frequently used models, such as linear and binary choice models, this condition reduces to the intensity of social interaction not being too large.

The solution of the equilibrium conditional expectations is the key to derive an efficient estimator. Except for some special cases, in general, there is not a closed-form solution. Therefore, much attention is paid to computation. We illustrate the solution process when privately known characteristics are discretely distributed with a finite support. When they are continuously distributed, since expectations are integrals, we approximate them by Gaussian quadratures. The values of expectation functions at fixed quadrature abscissae can be solved by contraction mapping and the expectation functions at the realized private information can be approximated by quadratures. The solution of equilibrium conditional expectations is nested in calculating sample log likelihood functions for the "nested fixed point" (NFXP) ML estimation, which was originated in Rust (1987) for the estimation of dynamic discrete choice models. The NFXP ML estimation performs well for both the linear and binary choice models. In Monte Carlo experiments, we also compare estimation under various information structures. It is found that estimation under a true information structure generally results in a higher maximized sample log likelihood than that under misspecified information structures. Hence, maximized log likelihood may provide a criterion for model selection. We also extend the model by incorporating unobserved group effects. The model is then applied to analyze peer's influence on smoking behaviors using the 2011 National Youth Tobacco Survey data.

The model in this paper is closely related to that in Blume et al. (2015), as both models have the two types of expectation heterogeneity. However, in their model, heterogeneity comes from the correlation of the econometrician-unobserved self-known shocks and the econometrician-observed publicly known individual characteristics. In our paper, heterogeneity arises from information asymmetry on the (econometrician-observed) exogenous characteristics. This framework is flexible to incorporate different types of information structures, not just public information and self-known variables, but also variables observed by agents on their linked friends. In addition, while Blume et al. (2015) adopt a quadratic utility function to derive a structural model for continuous choices, we make assumptions on expected utilities in a simultaneous move game under a general form of incomplete information and derive models for both continuous and discrete choices.

Both of our model and the model built by Blume et al. (2015) can be viewed as an extension to the classical framework in the literature of game estimation. For example, Bajari et al. (2010) investigate the estimation of a game under incomplete information, where a finite number of agents choose one action from a finite action set simultaneously. Private information resides in the i.i.d. idiosyncratic shock. All personal characteristics are exogenous and public information. Identification and estimation are based on the one-to-one correspondence between choice probabilities and expected payoffs. Incomplete information about both exogenous characteristics and idiosyncratic shocks is discussed in a two-player binary choice game in Aradillas-Lopez (2010). The author focused on one type of equilibrium where expectations of both players are based on a commonly observed public signal. As a result, expectations do not vary with asymmetric private information in that paper. For our model, when all the exogenous characteristics are public information, the information structure will become the same as that in Bajari et al. (2010). Xu (2012) builds a model for discrete choices with social interactions based on an incomplete information game with the same information structure as Bajari et al. (2010). Including linear models and binary choice models as special cases, the "linear-in-means" model of social interactions by Manski (1993) and Brock and Durlauf (2001) and the binary choice model by Lee et al. (2014) are all related to our model. In those models, the expected outcomes only depend on the publicly known characteristics. One interpretation may be that the expectations conditional on publicly known features are the expectations formed by the econometricians conditional on observed X. Our investigation suggests another interpretation. These expectations are also the expectations formed by individuals using their available information. Expectations do not change with self-information just because of independence and exogeneity.

Further related models are in Bisin et al. (2011) and Leung (2015). In Bisin et al. (2011), individual behaviors are socially interacted in the context that all exogenous characteristics are public information for group members and idiosyncratic shocks are privately known. In Leung (2015), endogenous formation of social networks is formulated as a game where agents choose social association simultaneously. In that model, whether an agent connects with another depends on individual features, pair-specific match value, and an idiosyncratic shock. Only the idiosyncratic shock is privately known. Discussions are focused on the case when individuals of identical attributes take the same strategies ex ante. Then Bayesian equilibrium reduces to a single function of exogenous characteristics. So our model is more general in terms of incomplete information.

The paper proceeds as follows. In Section 2, we introduce the model framework, as well as the relationship between the model and a simultaneous move game with incomplete information. In Section 3, we first show the one-to-one correspondence between a BNE and a consistent conditional expectation function and then discuss equilibrium existence and uniqueness. A detailed discussion on computation of equilibrium is in Section 4. We investigate identification and estimation in Sections 5 and 6. Monte Carlo experiments are conducted and some representative results are presented in Section 7. After an empirical analysis on smoking behaviors of adolescents in Section 8, we conclude in Section 9. Proofs of equilibrium analysis and numerical methods for equilibrium computation are put in Appendices A and B, respectively. Appendix C focuses on computations when some exogenous characteristics are known to socially connected agents and not others. Discussions on computations, identifications, and estimation for special cases are included in an appendix of a supplementary file (see Appendix D).[2]

2. Models

Consider a group of socially related agents of size n.[3] The $n \times n$ matrix, W, is used to represent their social or network relations. For

[2] The supplementary file is created due to page limitation on publication. In the supplementary file, there are details about the game theory structures, analytical equilibrium solution, and identification for the linear model of continuous choices and the binary choice model. There are elaborated discussions on estimation when there are group unobservables. Additional Monte Carlo experiments are also put in the appendix. The supplementary file is available online.

[3] Theoretically, we can analyze social interactions in a single group. In empirical application, however, we may have independent groups. In the presence of many groups for estimation, we will use subscript, g, as an index for a group.