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Directional distance functions: Optimal endogenous directions

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ABSTRACT

A substantial literature has dealt with the problem of estimating multiple-input and multiple-output production functions, where inputs and outputs can be good and bad. Numerous studies can be found in the areas of productivity analysis, industrial organization, labor economics, and health economics. While many papers have estimated the more restrictive output- and input-oriented distance functions, here we estimate a more general directional distance function. A seminal paper on directional distance functions by Chambers (1998) as well as papers by Färe et al. (1997), Chambers et al. (1998), Färe and Grosskopf (2000), Grosskopf (2003), Färe et al. (2005), and Hudgins and Primont (2007) do not address the issue of how to choose an optimal direction set. Typically the direction is arbitrarily selected to be 1 for good outputs and −1 for inputs and bad outputs. By estimating the directional distance function together with the first-order conditions for cost minimization and profit maximization using Bayesian methods, we are able to estimate optimal firm-specific directions for each input and output which are consistent with allocative and technical efficiency. We apply these methods to an electric-utility panel data set, which contains firm-specific prices and quantities of good inputs and outputs as well as the quantities of bad inputs and outputs. Estimated firm-specific directions for each input and output are quite different from those normally assumed in the literature. The computed firm-specific technical efficiency, technical change, and productivity change based on estimated optimal directions are substantially higher than those calculated using fixed directions.

1. Introduction

A large literature has dealt with the problem of estimating multiple-input and multiple-output production functions, where inputs and outputs can be good or bad. Numerous studies can be found in the areas of productivity analysis, industrial organization, labor economics, and health economics. Many studies in the area of child health estimate reduced form equations, thereby avoiding the direct estimation of disaggregated multiple-input, multiple-output structural equations, as summarized in Agee et al. (2012). Another area of extensive study has been firm efficiency, where some researchers assume an aggregate production technology, as in Fernandez et al. (2005).

Other researchers have directly estimated disaggregated multiple-input, multiple-output production functions (structural equations) using *distance* and *directional distance* functions in an attempt to measure the tradeoffs among inputs and outputs, without employing separability or aggregation assumptions. Using an output-oriented *distance function, the* researcher has two options. He can take the approach of holding inputs constant and scale bad outputs and good outputs by the same parameter to reach the production frontier. Pittman (1983) shows that this credits the firm for increasing a bad output (say pollution) along with a good output (say electricity). Alternatively, the researcher can hold constant bad outputs and inputs and measure the distance from the frontier using a proportional upward scaling of good outputs. However, no credit is given for a simultaneous reduction in bad outputs or inputs.

Similarly, using an input-oriented *distance function,* as with Atkinson and Dorfman (2005), a researcher has two options. First, he can hold constant good and bad outputs and scale back all inputs proportionally to reach the frontier. However, again no credit is given for any simultaneous increase (reduction) in good (bad) outputs. Alternatively, one can hold good outputs constant, treat bad outputs like inputs, and scale back both by the same factor of proportionality. However, the equal-proportionality assumption provides no credit for an increase in good outputs. None of these

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methods credits the firm for simultaneous changes in all inputs and outputs.

The less-restrictive *directional distance* function allows calculation of the distance to the production frontier using different directions for each input and output, both good and bad. If non-zero directions are used to change only inputs (outputs) when measuring productivity growth, the directional distance function is input- (output-) oriented. When non-zero directions are used for inputs and outputs, the directional distance function is technology-oriented. The choice of the direction of movement of a firm toward the frontier clearly affects measures of all structural parameters as well as the distance from a multiple-output production frontier. This becomes the basis for computing technical efficiency (TE), using an input-, output-, or technology-oriented approach, as well as productivity change (PC), which is the sum of the outward shift of the frontier, termed technical change (TC), and the extent to which the firm catches up to the frontier, termed efficiency change (EC). The latter equals the change in TE.

The exact direction chosen may substantially affect the calculation of TE as well as the measures comprising PC. As shown by Vardanyan and Noh (2006) and Agee et al. (2012), the parameter estimates depend on the choice of the directional vectors. However, a seminal paper on directional distance functions by Chambers (1998), as well as papers by Färe et al. (1997), Chambers et al. (1998), Färe and Grosskopf (2000), Grosskopf (2003), Färe et al. (2005), and Hudgins and Primont (2007) do not address the issue of how to choose an optimal direction set. As is typical with the empirical applications for each of these studies, Färe et al. (2005) estimated an output-oriented directional distance function for electric utilities involving three good inputs, one good output, and one bad output using directions of $+1$ for the good output and -1 for the bad output and all inputs. As a generalization of this approach, Agee et al. (2012) considered the impact of four distinct sets of directions on the estimated parameters of an output-oriented directional distance function, employed to explain measures of child health. While this provides information regarding sensitivity of model results to the directions chosen, the choice amongst them is left to the researcher. No framework has been provided within which to determine an optimal set of directions in a stochastic framework, although in a non-parametric framework Fare et al. (2013) make the choice of the direction vector endogenous.

Feng and Serletis (2014) propose a primal Divisia-type productivity index that incorporates undesirable outputs in a directional distance function with fixed directions of $(+1, -1)$ for good and bad outputs, respectively. However, the paper contains numerous restrictive assumptions, as indicated by Tsionas et al. (2014), which substantively affect results once they are generalized. Below we also discuss erroneous statements about required transformations for estimating distance and directional distance functions. We also note that the Feng and Serletis (2014) proposed aggregation index for productivity growth in their Eq. (4) is unit-sensitive and therefore is an improper aggregator of good and bad quantities.

In this paper we generalize the previous approaches by estimating the set of directions that is consistent with cost minimization and profit maximization. In order to accomplish this, we first formulate the restrictions that impose the fundamental translation property for input-, output-, and technology-oriented stochastic directional distance functions so that these restrictions contain the directions applied to each input and output. The translation property, akin to the property of linear homogeneity in input (output) quantities for an input (output) distance function, allows one to examine the effect of different directions of movement toward the frontier for different categories of inputs and outputs, both good and bad. We then generalize the dual relationship between the profit function and the technology-oriented directional distance function, as established by Chambers (1998), by assuming profit-maximizing behavior and deriving associated price equations for each input and output. These equations relate their prices to first-order partial derivatives of the directional distance function with respect to the quantity of each input and output and allow identification of directions for each input and output. This set of equations specializes to a system which models cost-minimizing behavior by utilizing only the associated price equations for each input.

We utilize our technique to model the electric utility production process using a set of inputs and outputs, both good and bad. Good inputs are energy (E), labor (L), and capital (K), which includes the annualized capital expenditures on environmental control for the two major restricted air pollutants, sulfur dioxide (SO_2) and nitrogen oxide (NO_X). These capital expenditures are for scrubbers to reduce SO_2 and NO_X emissions and/or modifications of combustion processes to reduce NO_X creation. We also include the sulfur content of fuels, S, as a bad input.[1] This generalization is important, since bad outputs can be reduced by switching to fuels with lower S, as well as modifying combustion processes or installing emission control devices. Trade-offs among these options have not been modeled in any previous study of electric utilities. Further, as bad outputs we include emissions of the three major pollutants—SO_2, carbon dioxide (CO_2), and NO_X. Since the emissions of CO_2 have never been regulated, typically, studies have included only SO_2 as a bad output. Good outputs are residential (R) and industrial/commercial (IC) electricity generation.

Using a panel of 77 US privately-owned firms producing steam-electric power over 10 years, we jointly estimate a quadratic technology-oriented directional distance function and a set of first-order conditions from the dual cost-minimization and profit-maximization models. The typical fixed-directions approach relies on the assumed directions of $(+1, -1)$ for good outputs and good inputs/bad outputs, respectively. However, we argue that since goods and non-marketed bads are produced by utilities, their relative valuation may not be 1-to-1 for all firms, when we model them as cost minimizers or profit maximizers. Since our data contain input and output price data, we append price equations (where prices are related to marginal products) for inputs to our directional distance function to obtain a cost-minimization directional distance system and the price equations for all good inputs and outputs to obtain a profit-maximization directional distance system. We identify the directions for bad inputs and bad outputs, which lack prices, using methods explained below. Using Markov Chain Monte Carlo (MCMC) methods we estimate these systems, obtaining estimates of all structural parameters, optimal directions, measures of TE, PC, TC, and EC, and estimates of the implied optimal percent changes in inputs and outputs. These directions are those that would prevail in the industry if firms were cost minimizers or profit maximizers. That is, we are estimating directional distance functions, not with directions chosen a priori, but with optimal directions chosen that are consistent with cost minimization or profit maximization.

As we show in our empirical application, the estimated optimal directions imply considerably larger measures of efficiency and productivity change than obtained using the fixed-directions approach. Optimal directions also imply that for the average firm to achieve cost minimization (i.e., be allocatively efficient), it must reduce K relative to L and E. To achieve profit maximization (assuming that pollutant emission levels are given), it must additionally reduce R and IC output.

[1] The study by Yaisawarng and Klein (1994) includes S, SO_2 emissions, electricity generation, and the required good inputs—production capital, fuel, and labor. However, they exclude the capital cost of pollution control equipment and the emissions of the other two major pollutants.