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Nonparametric instrumental variables estimation for efficiency frontier

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ABSTRACT

The paper investigates endogeneity issues in nonparametric frontier models. It considers a nonseparable model for a cost function $C = \varphi(Y, U)$ where C and Y are the cost and the output, U is uniform in $[0, 1]$ and φ is increasing with respect to U. The cost frontier corresponds to $U = 0$ and U can be interpreted as a normalized level of inefficiency. The endogeneity issue arises when Y is dependent of U. For identification and estimation, we use a nonparametric instrumental variables estimator of the model for fixed value $U = \alpha$, and obtain an estimate of the α-quantile cost frontier $\varphi(Y, \alpha)$. This involves the solution of a non linear integral equation. If the true frontier $\varphi(Y, 0)$ is wanted, it is then estimated by estimating the bias correction $\varphi(Y, 0) - \varphi(Y, \alpha)$ under additional regularity conditions. The procedure is illustrated through a simulated sample and with an empirical application to the efficiency of post offices.

1. Introduction

The important literature on production efficiency and frontier analysis has been essentially concentrated on the analysis of conditional distributions. Production models analyze the conditional distribution of the production (the output) given the inputs levels and cost frontier models consider the conditional distribution of the cost given the outputs. In these two cases, environmental variables may also be introduced but they are always treated as additional conditioning variables. This attention to conditional models is verified for parametric and nonparametric (DEA, FDH) models and for both deterministic and stochastic versions of the frontier. Some models consider conditional model to a given value (cost distribution given the output) and others are conditional to an inequality (the distribution of cost given the fact that the outputs are larger than some values). Both are conditional models.

Conditioning to some information is equivalent in the econometric literature to consider this information as exogenous information. Implicitly it is assumed that the process generating the conditioning elements does not contain any relevant information on the parameter of interest and that the conditional model identifies this parameter of interest.

In frontier analysis the exogeneity assumption has a particular meaning. Consider the case of a cost function, the exogeneity assumption means that the level of the outputs of a given firm is generated independently (or mean independently or non correlated) of the level of inefficiency of this firm. This property may be unrealistic in many situations. For instance, consider the case where a manager has to assign the quantity of outputs to produce to different production units. If the manager has some information on the level of inefficiency of each unit this may influence his choice (see e.g. Marschak and Andrews, 1944). Even if the manager is replaced by a population of consumers the demand may go to firms considered as being more efficient by the consumers. We will see below that treatment models allow to understand the mechanism driving endogeneity: in a cost model, the outputs are assigned by randomization but may contain bias selection. The same argument applies in production models where the quantity of inputs may suffer from bias selection. Our results will show that even in models where only inefficiency is present (no noise), DEA and FDH estimators can suffer from endogeneity.

The price to pay to relax the exogeneity assumption is that we need to extend the model. Our strategy belongs to the class of instrumental variables models. We will replace the conditioning of the cost to the outputs by conditioning to observed instruments. We also assume that these instruments explain sufficiently the endogenous variables to guarantee identifiability of the model.

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Alternative strategies may be used to address the endogeneity question: we may adopt a control function approach (see e.g. Imbens and Newey, 2009) where the introduction of a supplementary variable eliminates the endogeneity or we may adopt a more structural approach by the introduction of an explicit link between endogenous variables and inefficiency (see Simar et al., 2014). However the approach of the latter is quite different, Simar et al. analyze particular models where the endogeneity is introduced by some missing (unobserved) variables characterizing heterogeneity. In separable models the endogeneity problem may be addressed in two ways; by control functions (see Newey et al., 1999) or by instrumental variables (see Darolles et al., 2011). But to the best of our knowledge, the theory for the control function approach is not yet available for estimating quantile functions. In nonseparable models, which is our setup here, identification and estimation may be achieved by using nonparametric instrumental variables (IV) models.

So in this paper we will indeed concentrate our attention to nonseparable models satisfying some instrumental variables conditions. Basically if C is the cost and Y the outputs we analyze models defined by an equation $C = \varphi(Y, U)$ where U has a uniform in [0, 1] distribution independent from some instruments W and where φ is an increasing function of U. So U may be interpreted as a normalized inefficiency and the frontier is then equal to $\varphi(\cdot, 0)$; more generally the α-quantile of C is $\varphi(\cdot, \alpha)$. As explained below, for identification reasons, a direct estimation of $\varphi(\cdot, 0)$ is impossible. Then our strategy will be to estimate $\varphi(\cdot, \alpha)$ for some fixed values of α (this will require the solution of non linear integral equations), and then in a final step, to correct the bias between this quantile and the frontier. To the best of our knowledge, our paper is the first tentative to estimate a nonparametric frontier in the presence of endogeneity by applying nonseparable instrumental models.

Traditionally, parametric cost frontier models have often used models for the expectation of the cost and then by additional assumption on the error term, try to get estimates of the frontier (e.g., COLS, MOLS, etc.). It seems much more natural to concentrate the model on the full distribution of the cost described by its quantile function, small quantiles approaching the cost frontier. In this perspective, nonseparable models are quite natural, because they are based on the fact that a quantile function can be represented by a monotone transformation of a uniform variable U on [0, 1]. In addition, in the frontier setup, U is directly interpretable as the inefficiency. Separable models can be considered as a special case of nonseparable models.

The interest for our model is justified in Section 2 where the endogeneity in frontier models is presented in terms of treatment model. This clarifies what are exactly the issues of endogeneity in this particular setup. Section 3 is devoted to the nonparametric estimation of the α-quantile using the iterative resolution of a non linear integral equation. The bias correction for the estimation of the true function is analyzed in Section 4. In Section 5 we give some numerical illustrations of our method in a simulated example and in an empirical application. This allows to understand how to implement the estimator and its various components in practice. Section 6 concludes. Some technical details for the asymptotic properties of our estimator are displayed in the Appendix.

2. A treatment model for frontier analysis

One of the difficulties for the econometric analysis of endogeneity and for the understanding of its consequences is related to a correct definition of the parameter of interest. An important progress in this formalization has been realized in the context of treatment models which is based on the concept of counterfactual models, see e.g. Heckman and Vytlacil (2006). This concept allows the distinction between "fixing" the level of a variable and "conditioning" to the observation of this variable. For example a demand equation considers the reaction of the demand to any possible fixed level of price but we only observe a price generated by a market equilibrium and conditioning the demand by the observed price does not characterize the demand equation. The observation mechanism of the price creates endogeneity. Essentially we will say that we have an endogeneity problem if the parameter or the function of interest is not characterized by the assumed conditional distribution.

To simplify our presentation we consider a univariate cost model explained by a vector of outputs of dimension p. Also for simplification the model does not introduce additional environmental variables. The extensions to production functions is straightforward at least for the case of a single output and multiple inputs. As illustrated below, the concepts of exogeneity or endogeneity are easy to define if we adopt a presentation based on treatment models where the distinction between a counterfactual model and an observed model is introduced. This will be made in three steps: the counterfactual specification, an assignment mechanism and a process generating the observations.

2.1. The counterfactual specification

The first element of this model is the counterfactual specification. Let $\eta \in \mathbb{R}^p$ be a latent vector of the levels of the outputs. This multivariate index is non random and takes its value in all the possible values of the outputs which play here the role of the (continuous and multivariate) treatment. For each possible value of η there exists a cost, which is a random variable $C_\eta \geq 0$ and the (counterfactual) cost frontier is denoted by φ_η. It is defined as the minimum possible value of C_η. If the distribution of the random C_η is characterized by its cumulative distribution function F_η, or by its survivor function $S_\eta = 1 - F_\eta$, both assumed continuous, we have:

$$\begin{aligned}\varphi_\eta &= \inf\{c | F_\eta(c) > 0\} \\ &= \inf\{c | S_\eta(c) < 1\}. \end{aligned} \tag{2.1}$$

Let us underline that we have defined C_η as the cost corresponding to a value of the outputs exactly equal to η and not larger or equal to η, as is often done in the frontier literature (see Cazals et al., 2002). In the case of monotone frontier these two definitions are equivalent but the first one is necessary for our construction below.

Many models are possible for the definition of the cost distribution. In general the family $\left\{C_\eta\right\}_\eta$ may be viewed as a stochastic process indexed by η. We restrict the class of models by considering a single noise model based on the following nonseparable specification

$$C_\eta = \varphi_\eta(U) \tag{2.2}$$

where, for any η, $\varphi_\eta(\cdot)$ is a strictly increasing function of U and where, without loss of generality, U has a uniform distribution between 0 and 1.

We would like to point that this specification can be seen as being restrictive because the distribution of U is identical for any η, even if two units of production receiving two levels of η will have two different drawings of U, but from the same uniform distribution. We may imagine, as often the case in treatment models, more complex models with multivariate source of heterogeneity, like $C_\eta = \varphi_\eta(U_0, U_1, \ldots, U_r)$ for some $r > 1$, or models where U is replaced by a process U_η indexed by η. For example if η only takes a value in some finite set $\{1, \ldots, K\}$ we may imagine a vector $(U_1, \ldots, U_K)$ of noises and a relation $C_\eta = \varphi_\eta(U_\eta)$. We will not pursue these extensions here and we will develop our approach on the simple model (2.2), which stays in the usual framework of nonparametric frontiers, where the heterogeneity has only one random component U.

The specification (2.2) implies obviously that the frontier is given by

$$\varphi_\eta = \varphi_\eta(0). \tag{2.3}$$