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The effects of asymmetric volatility and jumps on the pricing of VIX derivatives

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ABSTRACT

This paper proposes a collection of affine jump–diffusion models for the valuation of VIX derivatives. The models have two distinctive features. First, we allow for a positive correlation between changes in the VIX and its stochastic volatility to accommodate asymmetric volatility. Second, upward and downward jumps in the VIX are separately modeled to accommodate the possibility that investors react differently to good and bad surprises. Using the VIX futures and options data from July 2006 through January 2013, we find conclusive evidence for both asymmetric volatility and upward jumps in VIX derivative prices. However, we find little evidence supporting downward jumps.

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1. Introduction

Over the past few years, there has been a great deal of financial innovation in volatility trading markets. A new collection of volatility derivatives, such as VIX futures, options, and exchange-traded products, has been introduced, making volatility trading more accessible to a broad range of investors.[1] The increasing volume of trading in those products is largely due to the fact that they can be used as a so-called tail risk hedging strategy against stock market downturns. That is, because changes in market volatility are negatively correlated with stock market returns, investors can limit the loss of an equity portfolio by taking a long position in VIX futures or call options. Given the explosive growth in volatility trading, the objective of this paper is to understand the effects of asymmetric volatility of and jumps in the VIX on the pricing of VIX futures and options.

We use the term "asymmetric volatility" to refer to the fact that the volatility of the VIX is not merely stochastic but also varies asymmetrically in response to changes in the VIX. Such asymmetry can be seen, for example, by looking at the dynamic relation between the VIX and the VVIX of the CBOE (Chicago Board Options Exchange). The VIX is a risk-neutral, forward-looking measure of market volatility implied by a cross section of S&P 500 index options, while the VVIX is a risk-neutral, forward-looking measure of market volatility of volatility implied by a cross section of VIX options.[2] The scatter plot in Fig. 1 shows that there is a positive relation between changes in the VVIX and in the VIX. That is, the volatility of the VIX, as measured by the VVIX, tends to increase (decrease) as the VIX increases (decreases).

An important implication of this empirical regularity is that the stochastic volatility factor implicit in the VIX options comprises two components—one that can be spanned by VIX futures and another that cannot. In other words, the VIX options market is nonredundant with the VIX futures market. To gauge the extent to which the options market can be spanned by the futures market, we run a regression of VVIX returns onto VIX returns, and find that the VIX changes can explain only 51% of the variation in the VVIX based on the adjusted R^2 of the regression.

Contrary to the empirical fact just discussed, the existing VIX option papers generally assume that the volatility of the VIX is either completely spanned or completely unspanned. For example,

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[1] See, for example, Whaley (2013) and Alexander and Korovilas (2012) as references on exchange-traded volatility products linked to the VIX.

[2] The VVIX is computed by applying the VIX formula to the VIX options market.

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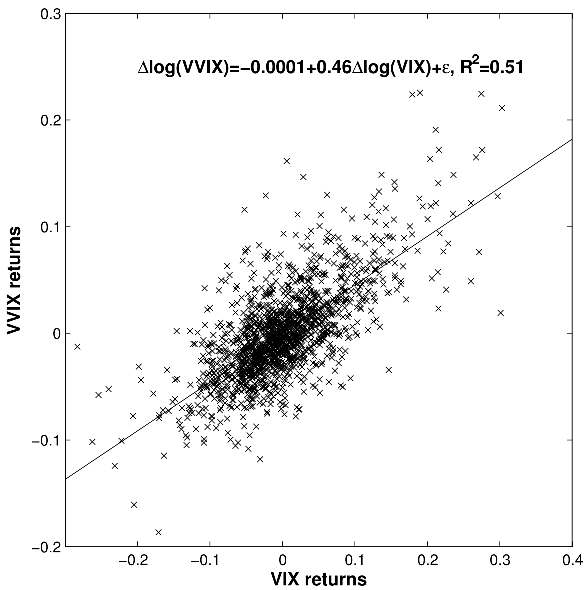

Fig. 1. Scatter plot of VVIX log returns against VIX log returns.

some authors, such as Grünbichler and Longstaff (1996), Detemple and Osakwe (2000), and Goard and Mazur (2013), apply fully spanned volatility models in which VIX options are completely hedgeable by VIX futures, whereas others, such as Mencía and Sentana (2012), introduce a completely unspanned volatility model in which innovations in the VIX index are uncorrelated with those in the volatility of the VIX index. It is evident that all of those models are unable to accommodate the important feature of the data that we have found—namely, asymmetric volatility. Hence, the chief goal of this paper is to contribute to the literature by studying the effects of asymmetric volatility on the pricing of VIX derivatives.

Intuitively, a model that allows for asymmetry in volatility may have great potential to improve the pricing of VIX options because it is capable of explaining the positive skewness implicit in the options.[3] The VIX options market shows a persistent deviation from a geometric Brownian motion regardless of time to maturity. In particular, out-of-the-money (OTM) VIX calls tend to have higher Black–Scholes implied volatility than OTM VIX puts, a tendency that is sometimes referred to as a volatility smile, implying that the option-implied VIX return distributions are positively skewed. This positive skewness may be in part attributable to asymmetric volatility.

On top of asymmetric volatility, we also look at the impact of jumps in the VIX on the valuation of VIX futures and options because they can be another channel for explaining the positive skewness implied by VIX options. Good and bad surprises may arrive with different rates and sizes and investors may react differently to them.[4] Hence, our models assume that upward and downward jumps occur independently with different frequencies and magnitudes. In this respect, our jump treatment is close in spirit to the currency option pricing model of Carr and Wu (2007), which separately models upward and downward jumps using time-changed Levy processes.

Building on the affine jump–diffusion framework of Duffie et al. (2000), we introduce a new family of dynamic models for the VIX with and without the features of asymmetric volatility and jumps and derive quasi-analytic solutions to the prices of futures and options. The models are characterized by three dynamic factors: one observed VIX index and two latent factors to capture time variations in the stochastic volatility and central tendency of the VIX. To reflect asymmetric volatility, we allow for a nonzero correlation between innovations in the VIX and in its stochastic volatility. Upward and downward jumps are both assumed to follow independent compound Poisson processes with each having its own jump intensity and exponential jump-size distribution. The models are then tested on the VIX futures and options data covering July 2006 through January 2013, via an unscented Kalman filter.

Turning to the empirical results, we first evaluate the effects of asymmetric volatility on the pricing of VIX derivatives. By comparing the model with symmetric volatility and no jumps (SVV) with the model with asymmetric volatility and no jumps (AVV), we find that the latter is strongly preferred to the former in both in-sample and out-of-sample tests. The performance difference is statistically significant at the 1% level based on the Diebold and Mariano (2002) test. That is, allowing for asymmetry in volatility can make large improvements in fitting the prices of VIX futures and options.

We next look at the effects of including upward jumps on top of asymmetric volatility by comparing the AVV model with the AVV-UJ model, the model with asymmetric volatility and upward jumps. The comparison shows decisive evidence in favor of the AVV-UJ model over the AVV model, except for in-sample futures pricing. The results are statistically significant at the 1% level according to the Diebold and Mariano (2002) test. That is, including upward jumps in the AVV model can make considerable improvements in VIX derivatives pricing.

Lastly, we investigate whether downward jumps can have an incremental effect on the pricing of VIX derivatives by comparing the AVV-UJ model with the model with asymmetric volatility and asymmetric upward and downward jumps (AVV-AJ). The comparison does not result in any significant ranking of one model over the other. Including downward jumps makes little, if any, improvement in the pricing of both futures and options as long as asymmetric volatility and upward jumps are accounted for. However, because we consider finite-activity jumps in this paper, we do not interpret our finding as implying that downward jumps are of no use in explaining VIX derivatives prices; rather, we argue that upward jumps play a far more important role in the pricing of VIX derivatives than downward jumps do.

Aside from the VIX option pricing literature, this paper also adds to the literature on unspanned stochastic volatility. Unspanned volatility has been found to be important in many other asset classes. For example, unspanned volatility in the interest rate market has been reported by Collin-Dufresne and Goldstein (2002), Li and Zhao (2006), and Bikbov and Chernov (2009), among others. Trolle and Schwartz (2009b) emphasize the importance of including unspanned volatility in pricing commodity options.

This paper is also related to the literature on the structure of jumps in the VIX. For example, Todorov and Tauchen (2011) and Wu (2011) both find that jumps in the VIX are more likely to have infinite activity, although they rely on different sources of information and estimation. More recently, Todorov et al. (2014) examine asymmetry in upward and downward jumps in VIX dynamics and find that upward and downward jumps are approximately symmetric, although the former are slightly more active than the latter. In more loosely related papers, Eraker (2005) and Broadie et al. (2007) study the impact of volatility jumps in joint analyses of stock index returns and options data. Our paper

3 This echoes a well-known finding that the negative skewness implicit in stock index options is partly associated with the leverage or volatility feedback effect, which indicates a negative correlation between stock index returns and volatility changes.

4 The market's asymmetric response to shocks or news announcements has been documented by Andersen et al. (2007) and Bakshi et al. (2008), among others.