Journal of Econometrics 196 (2017) 288–304

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

Identification and estimation of non-Gaussian structural vector autoregressions[☆]

Markku Lanne[a,b], Mika Meitz[a], Pentti Saikkonen[c,*]

[a] Department of Political and Economic Studies, University of Helsinki, P. O. Box 17, FI–00014 University of Helsinki, Finland

[b] CREATES, Denmark

[c] Department of Mathematics and Statistics, University of Helsinki, P. O. Box 68, FI–00014 University of Helsinki, Finland

ARTICLE INFO

Article history:
Received 15 April 2015
Received in revised form
12 January 2016
Accepted 6 June 2016
Available online 14 October 2016

JEL classification:
C32
C51
E52

Keywords:
Structural vector autoregressive model
Identification
Impulse responses
Non-Gaussianity

ABSTRACT

Conventional structural vector autoregressive (SVAR) models with Gaussian errors are not identified, and additional identifying restrictions are needed in applied work. We show that the Gaussian case is an exception in that a SVAR model whose error vector consists of independent non-Gaussian components is, without any additional restrictions, identified and leads to essentially unique impulse responses. Building upon this result, we introduce an identification scheme under which the maximum likelihood estimator of the parameters of the non-Gaussian SVAR model is consistent and asymptotically normally distributed. As a consequence, additional economic identifying restrictions can be tested. In an empirical application, we find a negative impact of a contractionary monetary policy shock on financial markets, and clearly reject the commonly employed recursive identifying restrictions.

1. Introduction

Vector autoregressive (VAR) models are widely employed in empirical macroeconomic research, and they have also found applications in other fields of economics and finance. While the reduced-form VAR model can be seen as a convenient description of the joint dynamics of a number of time series that also facilitates forecasting, the structural VAR (SVAR) model is more appropriate for answering economic questions of theoretical and practical interest. The main tools in analyzing the dynamics in SVAR models are the impulse response function and the forecast error variance decomposition. The former traces out the future effects of an economic shock on the variables included in the model, while the latter gives the relative importance of each shock for each variable. In order to apply these tools, the economic shocks (or at least the interesting subset of them) must be identified. Traditionally short-run and long-run restrictions, constraining the immediate and permanent impact of certain shocks, respectively, have been entertained, while recently alternative approaches, including sign restrictions and identification based on heteroskedasticity, have been introduced.

When SVAR models are applied, the joint distribution of the error terms is almost always (either explicitly or implicitly) assumed to have a multivariate Gaussian (normal) distribution. This means that the joint distribution of the reduced-form errors is fully determined by their covariances only. A well-known consequence of this is that the structural errors cannot be identified – any orthogonal transformation of them would do equally well – without some additional information or restrictions. This raises the question of the potential benefit of SVAR models with non-Gaussian errors whose joint distribution is not determined by the (first and) second moments only and which may therefore contain more useful information for identification of the structural shocks.

In this paper, we show that the Gaussian case is an exception in that SVAR models with (suitably defined) non-Gaussian errors are identified without any additional identifying restrictions. In the non-Gaussian SVAR model we consider, identification is

☆ The authors thank the Academy of Finland (grant number 268454), Finnish Cultural Foundation, and Yrjö Jahnsson Foundation for financial support. The first author also acknowledges financial support from CREATES (DNRF78) funded by the Danish National Research Foundation. Useful comments made by the Associate Editor and two anonymous referees have helped to improve the paper.

* Corresponding author.
E-mail addresses: markku.lanne@helsinki.fi (M. Lanne), mika.meitz@helsinki.fi (M. Meitz), pentti.saikkonen@helsinki.fi (P. Saikkonen).

http://dx.doi.org/10.1016/j.jeconom.2016.06.002

achieved by assuming mutual independence across the non-Gaussian error processes. The paper contains two identification results, the first of which allows the computation of (essentially) unique impulse responses. Identification is 'statistical' but not 'economic' in the sense that the resulting impulse responses and structural shocks carry no economic meaning as such; for interpretation, additional information is needed to endow the structural shocks with economic labels. Second, we obtain a complete identification result that facilitates developing an asymptotic theory of maximum likelihood (ML) estimation. A particularly useful consequence of this second result is that economic restrictions which are under-identifying or exactly-identifying in the conventional Gaussian set-up become testable. This is in sharp contrast to traditional identification approaches based on short-run and long-run economic restrictions which require the tested restrictions to be over-identifying (and finding even convincing exactly-identifying restrictions may be difficult). Moreover, sign restrictions, popular in the current SVAR literature, cannot be tested either (see, e.g., Fry and Pagan, 2011).

Compared to the previous literature on identification in SVAR models exploiting non-Gaussianity, our approach is quite general. Similarly to us, Hyvärinen et al. (2010) and Moneta et al. (2013) also assume independence and non-Gaussianity, but, in addition, they impose a recursive structure, which in our model only obtains as a special case. Lanne and Lütkepohl (2010) assume that the error term of the SVAR model follows a mixture of two Gaussian distributions, whereas our model allows for a wide variety of (non-Gaussian) distributions. Identification by explicitly modeling conditional heteroskedasticity of the errors in various forms, considered by Normandin and Phaneuf (2004), Lanne et al. (2010), and Lütkepohl and Netšunajev (2014b), is also covered by our approach. In fact, identification by unconditional heteroskedasticity (see, e.g., Rigobon, 2003) is the only approach in the previous literature we do not cover.

We apply our SVAR model to examining the impact of monetary policy in financial markets. There is a large related literature that for the most part relies on Gaussian SVAR models identified by short-run restrictions. While empirical results vary depending on the data and identification schemes, typically a monetary policy shock is not found to account for a major part of the variation of stock returns. This is counterintuitive and goes contrary to recent theoretical results (see Castelnuovo, 2013 and the references therein). Our model, with the errors assumed to follow independent Student's t-distributions, is shown to fit recent U.S. data well, and we find a strong negative, yet short-lived, impact of a contractionary monetary policy shock on financial conditions, as recent macroeconomic theory predicts. Moreover, the recursive identification restrictions employed in much of the previous literature are clearly rejected.

The rest of the paper is organized as follows. In Section 2, we introduce the SVAR model. Section 3 contains the identification results. First we show how identification needed for the computation of impulse responses is achieved and then how to obtain complete identification needed in Section 4 where we develop an asymptotic estimation theory and establish the consistency and asymptotic normality of the maximum likelihood (ML) estimator of the parameters of our model. In addition, a three-step estimator is proposed that may be useful in cases where full ML estimation is cumbersome due to short time series or the high dimension of the model. As both estimators have conventional asymptotic normal distributions, standard tests (of, e.g., additional economic identifying restrictions) can be carried out in the usual manner. An empirical application to the effect of U.S. monetary policy in financial markets is presented in Section 5, and Section 6 concludes.

Finally, a few notational conventions are given. All vectors will be treated as column vectors and, for the sake of uncluttered notation, we shall write $x = (x_1, \ldots, x_n)$ for the (column) vector x where the components x_i may be either scalars or vectors (or both). For any vector or matrix x, the Euclidean norm is denoted by $\|x\|$. The vectorization operator $vec(A)$ stacks the columns of matrix A on top of one another. Kronecker and Hadamard (elementwise) products of matrices are denoted by $\otimes$ and $\odot$, respectively. Notation $\imath_i$ is used for the ith canonical unit vector of $\mathbb{R}^n$ (i.e., an n-vector with 1 in the ith coordinate and zeros elsewhere), $i = 1, \ldots, n$ (the dimension n will be clear from the context). An identity matrix of order n will be denoted by I_n.

2. Model

Consider the structural VAR (SVAR) model

$$y_t = \nu + A_1 y_{t-1} + \cdots + A_p y_{t-p} + B\varepsilon_t, \tag{1}$$

where y_t is the n-dimensional time series of interest, ν $(n \times 1)$ is an intercept term, $A_1, \ldots, A_p$ and B $(n \times n)$ are parameter matrices with B nonsingular, and ε_t $(n \times 1)$ is a temporally uncorrelated strictly stationary error term with zero mean and finite positive definite covariance matrix (more specific assumptions about the covariance matrix will be made later). As we only consider stationary (or stable) time series, we assume

$$\det A(z) \overset{def}{=} \det\left(I_n - A_1 z - \cdots - A_p z^p\right) \neq 0, \quad |z| \leq 1 \ (z \in \mathbb{C}). \tag{2}$$

Left-multiplying (1) by the inverse of B yields an alternative formulation of the SVAR model,

$$A_0 y_t = \nu^{\bullet} + A_1^{\bullet} y_{t-1} + \cdots + A_p^{\bullet} y_{t-p} + \varepsilon_t, \tag{3}$$

where ε_t is as in (1), $A_0 = B^{-1}$, $\nu^{\bullet} = B^{-1}\nu$, and $A_j^{\bullet} = B^{-1}A_j$ $(j = 1, \ldots, p)$. Typically the diagonal elements of A_0 are normalized to unity, so that the model becomes a conventional simultaneous-equations model. In this paper, we shall not consider formulation (3) in detail.

The literature on SVAR models is voluminous (for a recent survey, see Kilian (2013)). A central problem with these models is the identification of the parameter matrix B: without additional assumptions or prior knowledge, B cannot be identified because, for any nonsingular $n \times n$ matrix C, the matrix B and the error term ε_t in the product $B\varepsilon_t$ can be replaced by BC and $C^{-1}\varepsilon_t$, respectively, without changing the assumptions imposed above on model (1). This identification problem has serious implications on the interpretation of the model via impulse response functions that trace out the impact of economic shocks (i.e., the components of the error term ε_t) on current and future values of the variables included in the model. Impulse responses are elements of the coefficient matrices $\Psi_j B$ in the moving average representation of the model,

$$y_t = \mu + \sum_{j=0}^{\infty} \Psi_j B\varepsilon_{t-j}, \quad \Psi_0 = I_n, \tag{4}$$

where $\mu = A(1)^{-1}\nu$ is the expectation of y_t and the matrices Ψ_j $(j = 0, 1, \ldots)$ are determined by the power series $\Psi(z) = A(z)^{-1} = \sum_{j=0}^{\infty} \Psi_j z^j$. As the preceding discussion makes clear, for a meaningful interpretation of such an analysis, an appropriate identification result is needed to make the two factors in the product $B\varepsilon_t$, and hence the impulse responses $\Psi_j B$, unique.

So far we have only made very general assumptions about the SVAR model, implying uniqueness only up to linear transformations of the form $B \to BC$ and $\varepsilon_t \to C^{-1}\varepsilon_t$ with C nonsingular. In SVAR models of the type (1), the covariance matrix of the error term is typically restricted to a diagonal matrix so that the transformation matrix C has to be of the form $C = DO$ with O orthogonal