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Testing for central dominance: Method and application

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ABSTRACT

Central dominance (CD) introduced in Gollier (1995, *Journal of Economic Theory*) is a risk concept that differs from stochastic dominance (SD) in an important way. In particular, CD implies a deterministic comparative static of a change in decision when risk changes, but SD does not have such an implication. In this paper, we propose the first test of central dominance, which amounts to checking a functional inequality. We derive the asymptotic distribution of a lower bound of the proposed test and suggest a bootstrap procedure to compute the critical values. We also conduct simulations to evaluate the performance of this test. Our empirical study finds clear evidence of CD relations between the S&P 500 index return distributions during 2001–2013 and results in unambiguous implications for investment decisions.

1. Introduction

A major building block of modern risk theory is the notion of stochastic dominance (SD) introduced in Hadar and Russell (1969), Hanoch and Levy (1969), and Rothschild and Stiglitz (1970); see Levy (1992) for a survey. It is well known that SD can determine a preference ordering of different risks. In particular, Rothschild and Stiglitz (1970) show that all risk-averse agents, i.e., those with increasing and concave utility functions, prefer risk A to risk B if, and only if, the distribution associated with A second-order stochastically dominates that of B. Yet, SD does not imply a change in demand when risk changes. For example, Rothschild and Stiglitz (1971) find that risk-averse agents need not reduce their demand for a risky asset when its risk increases in the sense of second-order SD; also see Eeckhoudt and Gollier (2000) for a numerical example. Thus, SD offers limited practical direction for adjusting investment decisions after the distribution of risk changes.

Among many researchers that try to link risk and demand directly,[1] Gollier (1995) makes an important contribution by introducing the new concept, "central dominance" (CD), and shows that risk-averse investors demand less of a risky asset if, and only if, its risk increases (the associated distribution being dominated) in the sense of CD. Hollifield and Kraus (2009) further elaborate on this idea and analyze the condition under which a demand-reducing change in risk makes all risk-averse investors worse off. It must be emphasized that, while CD implies a deterministic comparative static of a change in decision when the risk (distribution) changes, SD does not have a similar implication. It has also been shown that second-order SD is neither sufficient nor necessary for CD (Gollier, 1995).

Despite the practical relevance of CD, testing CD has not been considered in the literature, to the best of our knowledge. According to Gollier (1995), CD is defined as the existence of some parameter such that a functional inequality holds; yet, it is not easy to construct a test for an inequality constraint. The study of CD has been limited partly because there has been no test of CD available.[2] This paper intends to fill this gap and proposes a test of CD. We first transform the functional inequality in the definition of CD into an equality and then construct a test on this equality condition based on the maximum of an integral process.[3] We derive the asymptotic

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1 See, e.g., Sandmo (1971), Eeckhoudt and Hansen (1980), Meyer and Ormiston (1985), Black and Bulkley (1989), Landsberger and Meilijson (1990), Dionne and Gollier (1992), Eeckhoudt and Gollier (1995), Gollier (1995, 1997), and Tzeng (2001).

2 The study of SD suffers from a similar difficulty. Note that testing SD, which also requires checking an inequality constraint, has received more attention only recently; see, e.g., Anderson (1996), Davidson and Duclos (1997, 2000), Barrett and Donald (2003), Linton et al. (2005), Horváth et al. (2006), Bennett (2007), Linton et al. (2010), and Donald and Hsu (2016).

3 Chen and Szroeter (2009) and Linton et al. (2010) also construct tests by transforming moment inequalities into equalities.

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distribution of the proposed test and suggest a bootstrap procedure to compute the critical values. Simulations are then conducted to evaluate the performance of this test.

In the empirical study, we apply the proposed test to the daily return distributions of the S&P 500 index from 2001 to 2013. Our empirical study finds clear evidence of CD relations during that period of time and results in unambiguous implications for investment decisions. We find, for example, that the return distributions in 2003, 2004, 2006 and 2010 centrally dominate, respectively, those in 2004, 2005, 2007 and 2011. These findings suggest that the optimal investment amounts in 2004, 2005, 2007 and 2011 should be lower than what they were in the previous year. We also find that the return distributions in 2006, 2010, 2012 and 2013 centrally dominate, respectively, those in 2005, 2009, 2011 and 2012, so that the optimal investment amounts in 2006, 2010, 2012 and 2013 should be higher than what they were in the previous year.

This paper is organized as follows. In Section 2, we review the conditions and properties of CD; examples are also provided to illustrate the difference between SD and CD. In Section 3, we introduce the proposed test and establish its asymptotic properties. Monte Carlo simulation results are reported in Section 4. An empirical study on S&P 500 index return distributions based on the proposed test is presented in Section 5. Section 6 concludes the paper. All proofs are collected in the Appendix.

2. Central dominance

2.1. Theory

Consider a representative agent who faces the optimal decision problem with respect to a change in risk. We follow the setup in Gollier (1995), and make the following assumptions.

Assumption 1. Assume that:

1. The individual has an increasing, concave, and twice differentiable von Neumann–Morgenstern utility function $u(z(\alpha, x))$, where $z(\alpha, x)$ is a payoff function.

2. The payoff of the individual has the form $z(\alpha, x) = \alpha x + z_0$, which is determined by a decision variable α and a risk variable x, where z_0 is an exogenous parameter.

3. The range of α is normalized to [0, 1]. The random variable x is defined on $[a, b]$ with $a < 0 < b$ and has a continuous distribution function F with $E[x] > 0$.[4]

The first condition of Assumption 1 ensures that the individual is risk averse. In the second condition, we choose a particular form of the payoff function which entails the standard portfolio problem, the problem of a competitive firm with a constant marginal cost, and the insurance problem. For more details and examples, see Gollier (1995). Note that the third condition is required to avoid a boundary solution for α.

When the distribution function F is known to the individual, he/she chooses the optimal $\alpha^*(u; F)$ to maximize his/her expected utility. The following proposition due to Gollier (1995) gives a deterministic change in the optimal decision after a certain change in risk.

Proposition 2.1. *All individuals have their $\alpha^*(u; F) \geq \alpha^*(u; G)$ after the change in the risk distribution from F to G if, and only if, there exists $\gamma \in \mathbb{R}$ such that*

$$\gamma T(x; F) \geq T(x; G), \text{ for all } x \in [a, b]. \tag{1}$$

Here $T(x; F) = \int_a^x tdF(t)$, and $T(x; G) = \int_a^x tdG(t)$.

Proposition 2.1 provides a necessary and sufficient condition, hereafter Condition (1), for *all* individuals to decrease their decision variable after a change in risk from distribution F to G. When Condition (1) holds, we say that F centrally dominates G, denoted as $F \overset{CD}{\succ} G$.[5]

Since

$$T(x; F) = \int_a^x tdF(t) = E_F[t|t \leq x]F(x),$$

$T(x; F)$ could be viewed as the conditional expectation of t given $t \leq x$ multiplied by the probability of $t \leq x$. It follows that Condition (1) can be rewritten as:

There exists a real number γ satisfying

$$\gamma E_F[t|t \leq x]F(x) \geq E_G[t|t \leq x]G(x), \text{ for all } x \in [a, b].$$

This is a continuum of the conditional moment inequality plus an existence condition.

2.2. Example: Investment decision

CD and SD are two distinct concepts. CD implies a deterministic change in the optimal decision variable, but SD does not have similar implications. The following example illustrates that SD and CD do not imply each other.

Consider a traditional portfolio problem: there are two assets in the market, one is risk free with the rate of return r_f and the other is risky with the rate of return y, where $y \in [\underline{y}, \bar{y}]$. An investor with initial wealth W chooses to invest α in the risky asset. The final wealth of this individual is then

$$\begin{aligned} \alpha(1+y) + (W-\alpha)(1+r_f) &= \alpha(y-r_f) + W(1+r_f) \\ &= \alpha x + z_0, \end{aligned}$$

where $x = y - r_f$ is the excess return and $z_0 = W(1 + r_f)$.

Let F and G represent two distributions of the excess return with $F \overset{CD}{\succ} G$, which means that there exists a real number γ satisfying

$$\gamma \int_a^x tdF(t) \geq \int_a^x tdG(t), \text{ for all } x \in [a, b],$$

where $a = \underline{y} - r_f$ and $b = \bar{y} - r_f$. In addition, let $u(\cdot)$ be an increasing and concave utility function of the investor. The objective of the investor under distribution F is to choose an α to maximize the expected utility:

$$E_F\left[u\left(\alpha x + z_0\right)\right]. \tag{2}$$

Thus, the first-order condition of the problem (2) can be written as

$$E_F\left[xu'\left(\alpha x + z_0\right)\right] = 0.$$

By integration by parts, the first-order condition can be further rewritten as

$$u'\left(\alpha b + z_0\right) T(b; F) - \int_a^b \alpha u''\left(\alpha x + z_0\right) T(x; F) dx = 0. \tag{3}$$

[4] The assumption of bounded support for x, while ruling out unbounded distributions, is made for simplicity. Although this is a limitation of our result, we note that similar conditions are also frequently adopted in testing functional inequalities, such as tests of SD, see, e.g. Barrett and Donald (2003) and Donald and Hsu (2016).

[5] In Gollier (1995), it is stated that "G is centrally riskier than F" and is denoted as F CR G when Condition (1) holds. For more discussions, examples, and illustrations about this proposition, see Gollier (1995).