Journal of Econometrics 191 (2016) 302–311

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

On independence conditions in nonseparable models: Observable and unobservable instruments

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ARTICLE INFO

Article history:
Available online 22 January 2016

ABSTRACT

This paper develops identification results employing independence conditions among unobservable variables. The independence conditions are used to derive first-stage nonseparable reduced form functions. Once constructed, these reduced form functions are employed to express the derivatives of nonseparable structural functions in terms of the derivatives of the reduced form functions. For models with simultaneity, we obtain the new results by combining the independence assumptions together with parametric specifications and exclusion restrictions. For models with triangularity, we allow all functions to be nonparametric and nonseparable in unobservable random terms. For the latter, we provide several equivalence results and discuss some of the trade-offs between observable and unobservable instruments.

1. Introduction

We consider the use of independence conditions among unobservable variables as a source of identifying restrictions in nonparametric models with endogeneity. We show how to exploit the independence conditions to construct first-stage reduced form functions, which we then use to identify the derivatives of nonparametric, nonseparable functions of interest. We develop the methods for two semiparametric models with simultaneity and for a nonparametric triangular model.

In linear simultaneous equation models with additive unobservable random terms, identification is usually achieved by exclusion restrictions, equality restrictions, or covariance restrictions (see Hausman (1983) and Hsiao (1983) for surveys). In nonparametric models with simultaneity, the most commonly used methods proceed by employing observable variables that are excluded from the equations of interest, correlated with the endogenous variables, and independent of unobservable random terms. This limits the possibilities when such instruments are not observed. The methods that we develop can be considered as possible alternatives in such cases, where instruments that can be used to identify a function of interest are not observed. The model then becomes one with independence restrictions among unobservable variables. We consider triangular models with all equations in the system being nonparametric and nonseparable in unobservable random terms, and simultaneous equation models where the function of interest is nonparametric and nonseparable in an unobservable random term, while the other equations in the system are linear in parameters and additive in unobservable random terms. The specific restrictions that we impose in the simultaneous equations models allow us to derive first-stage reduced form functions without identifying first the structural function of interest. We can then use those identified reduced form functions in the same way as in triangular models, to identify the derivatives of the function of interest.

The approach developed in this paper to identify triangular and simultaneous equations models is an extension of Chesher's (2003) approach. Chesher (2003) showed how to constructively identify the derivatives of a nonparametric and nonseparable function in a triangular model by estimating first stage conditional quantile functions, given exogenous variables, for the endogenous variables in the equation of interest. The derivatives of the function of interest were then obtained as functions of the derivatives of the conditional quantile functions.[2] Instead of using observable instruments, as in Chesher (2003), we use unobservable ones. We extend Chesher's approach to identify functions in simultaneous equations by identifying reduced form functions using Matzkin (2003, 2008, 2010, 2015).

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[1] Sections 2 and 5 are based on results previously circulated in the working paper entitled "Unobservable Instruments".

[2] Previously, using conditional expectations rather than conditional quantiles, Newey et al. (1999) employed a similar approach to identify a separable nonparametric function in a triangular model.

http://dx.doi.org/10.1016/j.jeconom.2015.12.003
0304-4076/

For models with simultaneity, other approaches can also be used to exploit independence among the unobservable random terms in a system. Two possible such alternative methods are the conditional density method (Matzkin, 2007a, 2008, 2010, 2012, 2013, 2015; Berry and Haile, 2014, 2015; Blundell et al., 2013) and nonparametric IV (Newey and Powell, 2003; Ai and Chen, 2003; Chernozhukov and Hansen, 2005; Hall and Horowitz, 2005; Darolles et al., 2011; Chen and Pouzo, 2012; Chen et al., 2014, and several more recent works). Applying these methods would usually require a different set of assumptions than the ones we use in this paper.

We consider two models with simultaneity. The first model is a two equation system. In this system, the equation of interest has no excluded observable variables, but it includes as one of its arguments an observable variable that can be used as an instrument to identify the second equation. This allows one to identify the unobservable random term of the second equation, which is then used as an instrument to identify the equation of interest. Hausman and Taylor (1983) showed identification of the functions in such system, through covariance restrictions, when the first and second functions were linear in parameters and additive in the random terms. We allow the function of interest to be nonparametric and nonadditive in the unobservable random term, and show how it can be identified using the distribution of observable and identified unobservable variables. As an application, suppose we are interested in estimating the market demand function for a product. We observe income of the consumers but possess no observations on input prices or other variables that could shift the supply equation. The assumption that those unobserved variables are independent of the unobservable in the demand equation would in many cases be satisfied. In such cases, we could use the method in this paper to identify an unobservable variable representing the effect of those unobservable supply shifters, and use them to identify demand using conditional distribution functions.

Our second model is one with three endogenous variables and no observable exogenous variables. We use the mutual independence of the unobservable random terms together with exclusion restrictions in the equations and functional restrictions in two of the equations to identify a nonadditive function of interest. The model is a nonparametric extension of the last example in Hausman and Taylor (1983). In that example, the first endogenous variable is a linear function of a second endogenous variable and a random term, the second endogenous variable is a linear function of a third endogenous variable and another random term, and the third endogenous variable is a linear function of the first endogenous variable and another random term. The model is suitable for analyzing, for example, the actions of members in a group, where each one is influenced by a different member of the group. We keep the linear assumptions on two of the functions but let the third function be nonparametric and nonadditive. We adapt the methods in Matzkin (2010, Section 4; 2015, Section 4) to identify the unobservable random terms of the linear equations, employing expressions for the unknown parameters in the linear functions in terms of logs of the density of the observable variables. (Matzkin (2016) presents more extensive and general results for identification of parameters in models with simultaneity.) We then use these two unobservable random terms to construct reduced-form functions for the endogenous variables that are used to identify the nonseparable function of interest.

Our results for triangular models do not impose either parametric or separability conditions. We show that conditional independence among the unobservable variables in the system, at a unique value of the conditioning variable, provides identification of the derivatives of the nonparametric, nonseparable function of interest. An alternative, commonly used, identification method when independence conditions among the unobservable variables in the system are not satisfied, is the control function approach (Heckman and Robb, 1985). In nonseparable models, this requires an observable variable that is independent of the vector of unobservable random variables in the system. This observable instrument is used to identify an unobservable variable, excluded from the equation of interest. The identified unobservable variable is then used as an additional conditioning variable in the identification of the equation of interest. The control function approach has been used in semiparametric and nonparametric models by Ng and Pinkse (1995), Newey et al. (1999), Pinkse (2000), Chesher (2003), and Imbens and Newey (2009), among others. Florens et al. (2008) consider identification of average treatment effects in models with continuous endogenous variables using this approach, among others. Ma and Koenker (2006) developed quantile regression estimation methods (originally developed in Koenker and Bassett (1978)). Blundell and Powell (2004) used this approach to develop estimation methods for a semiparametric binary response model. (See Blundell and Powell (2003) for a survey of these and other methods used in nonparametric and semiparametric regression models. See also Rothe (2009) for a more recent method for estimating semiparametric binary response models using control functions.) Although very useful for triangular models when an instrument is observed, the control function approach can be employed for identification of models with simultaneity only in very specific cases. (See Blundell and Matzkin (2014).) More recent work on triangular models with observable instruments include Torgovitsky (2015) and D'Haultfoeuille and Fevrier (2015), who allow the observable instrument to be discrete. Also for triangular models, Altonji and Matzkin (2005) provide estimators for the structural function and distribution, and for average derivatives, when the conditioning variable is endogenous, either continuously distributed or discrete.

The structure of the paper is as follows. In the next section we discuss triangular models, constructions of first-stage reduced form functions, and trade-offs between observable and unobservable instruments. In Sections 3 and 4 we present the results for models with simultaneity. In Section 5 we provide equivalence results for identification in triangular models, and discuss applications to binary response models. Section 6 concludes.

2. Identification using first-stage reduced form functions

The objective of this section is to discuss previous results on estimation of nonparametric nonseparable models, to show how they can be used to construct first-stage reduced form functions, and to demonstrate in a triangular model how these can be used to identify the derivatives of a function of interest.

Consider first a model where

$$Y_1 = m(X, \varepsilon_1) \tag{2.1}$$

and where the unknown function m is strictly increasing in ε_1. If X and ε_1 are distributed independently and if the unknown distribution F_{ε_1} of ε_1 is strictly increasing, then by Matzkin (2003) it follows that for all x, ε_1 at which the conditional distribution of Y_1 given $X = x$ is defined,

$$m(x, \varepsilon_1) = F^{-1}_{Y_1|X=x}\left(F_{\varepsilon_1}(\varepsilon_1)\right). \tag{2.2}$$

In other words, the function m is identified, up to a monotone transformation of ε_1, from the distribution of (Y_1, X). The invertibility of m in ε_1 implies that given (y_1, x) there exists a unique value of ε_1 satisfying (2.1). Assuming differentiability, the partial derivative, $\partial m(x, \varepsilon_1)/\partial x$, of m with respect to x, when the value of ε_1 remains fixed at such value, is given by

$$\frac{\partial m(x, \varepsilon_1)}{\partial x} = \frac{-\frac{\partial F_{Y_1|X=x}(y_1)}{\partial x}}{\frac{\partial F_{Y_1|X=x}(y_1)}{\partial y_1}} = \frac{-\frac{\partial F_{Y_1|X=x}(y_1)}{\partial x}}{f_{Y_1|X=x}(y_1)}.$$