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Impulse response matching estimators for DSGE models

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ABSTRACT

The existing asymptotic theory for VAR-based impulse response matching estimators of the structural parameters of DSGE models does not cover situations in which the number of impulse responses exceeds the number of VAR model parameters. We establish the consistency of the estimator in this situation, we derive its asymptotic distribution, and we show how this distribution can be approximated by bootstrap methods. We also demonstrate that under our assumptions special care is needed to ensure the asymptotic validity of Bayesian methods of inference. Finally, we show how to deal with weak identification both under our assumptions and under standard assumptions.

1. Introduction

Structural impulse responses play a central role in modern macroeconomics. It is common to estimate the structural parameters of a dynamic stochastic general equilibrium (DSGE) model by choosing these parameters so as to minimize a suitably weighted average of the distance between the structural impulse responses implied by the DSGE model and the corresponding structural impulse responses implied by an approximating vector autoregressive (VAR) model fit to actual data. One advantage of this approach compared with full information maximum likelihood estimators of DSGE models is that it does not require the model to fit well in all dimensions, but allows the user to focus on the dimension of the model that matters most to macroeconomists (also see Dridi et al., 2007; Hall et al., 2012).

Such impulse response matching estimators have been employed in Rotemberg and Woodford (1997), Christiano et al. (2005), Iacoviello (2005), Boivin and Giannoni (2006), Uribe and Yue (2006), DiCecio and Nelson (2007), Dupor et al. (2007), Jordà and Kozicki (2011), DiCecio (2009), and Altig et al. (2011), among others. In related research, Christiano et al. (2011) propose a Bayesian version of the impulse response matching estimator in which the quasi-likelihood function based on the distance between VAR and DSGE model impulse responses is combined with prior information. Other applications of Bayesian impulse response matching estimators include Christiano et al. (2016) and Kormilitsina and Nekipelov (2016).

Because impulse response matching estimators are classical minimum distance (CMD) estimators, by construction they inherit the usual properties of CMD estimators (see, e.g. Newey and Smith, 2004). Notably, the use of the optimal weighting matrix induces finite-sample bias in the estimator, which is why most applied users employ a diagonal weighting matrix instead. In this paper we identify another potential problem that is specific to impulse response matching estimators. In estimating the structural parameters of DSGE models, macroeconomists often match response functions evaluated across many horizons such that the number of impulse response coefficients exceeds the dimensionality of the VAR model parameters (see, e.g. Iacoviello, 2005; Uribe and Yue, 2006; Altig et al., 2011). This practice causes the asymptotic covariance matrix of the structural impulse responses to be singular,

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which in turn renders the asymptotic behavior of the resulting impulse response matching estimator nonstandard. As a result, standard asymptotic and finite-sample results for CMD estimators no longer apply. We develop an alternative asymptotic theory of the impulse response matching estimator for this practically relevant context. Our paper makes four distinct theoretical contributions.

First, we show that in this case the impulse response matching estimator has a nonstandard convergence rate when using the optimal weighting matrix. While the estimator remains consistent, its asymptotic distribution is nonstandard. Both the rate of convergence and the nonnormality of the asymptotic distribution differ from standard results for CMD estimators. We establish that the nonstandard asymptotic approximation may be recovered by bootstrap methods. Of course, in the absence of asymptotic normality, one would not want to report standard errors for this estimator, but rely on bootstrap confidence intervals that do not rely on asymptotic normality.

In contrast, the impulse response matching estimator based on the diagonal weighting matrix remains $\sqrt{T}$-consistent and asymptotically normal as in the standard CMD case. Its asymptotic variance may be approximated by the same bootstrap methods as in the case in which the dimensionality of the impulse response vector is no larger than that of the VAR model parameters. The latter result provides a formal justification for the use of the diagonal weighting matrix in applied work in a case not covered by existing asymptotic theory.

Second, our asymptotic results matter not only for the construction of point and interval estimates for structural parameters. We also prove that conventional tests of overidentifying restrictions, as employed in Boivin and Giannoni (2006), for example, have a nonstandard asymptotic distribution, when the number of impulse response parameters exceeds the number of VAR model parameters, invalidating the use of conventional critical values.

Third, our work also has implications for the use of Bayesian impulse response matching estimators. Often in the literature, Bayesian estimators are used as a convenient device for constructing asymptotic approximations. It may be tempting to base inference on a point estimate constructed from the mean, median or mode of the quasi-posterior of the structural parameters together with an estimate of the asymptotic standard error based on the standard deviation of this distribution. Although Markov Chain Monte Carlo methods may indeed be used to construct point estimators of the structural parameters based on the mean, median or mode, we show that one cannot use the standard deviations of the quasi-posterior distribution to approximate the asymptotic standard errors of the structural parameter estimator, when the number of impulse responses exceeds the number of VAR model parameters. This is true whether one employs the optimal weighting matrix or the diagonal weighting matrix. We show that asymptotically valid Bayesian inference may be conducted by constructing the variance using the sandwich formula of Chernozhukov and Hong (2003), however.

Fourth, it is well known that structural parameters of macroeconomic models may not be strongly identified. This problem also afflicts impulse response matching estimators, as documented in Canova and Sala (2009). We propose a nonstandard confidence interval for the structural parameters of the underlying data generating process that is robust to weak identification problems.

The remainder of the paper is organized as follows. Section 2 establishes the consistency of the impulse response matching estimators in question, derives their asymptotic distribution, and proposes suitable bootstrap methods of inference. Because both the impulse response matching estimator based on the optimal weighting matrix and the estimator based on the diagonal weighting matrix are practically feasible and asymptotically valid, the question arises which approach implies more accurate confidence intervals for structural model parameters in finite samples.

Section 3 evaluates the quality of these asymptotic approximations based on a Monte Carlo simulation experiment. Based on a small-scale New Keynesian model we provide some tentative evidence that confidence intervals for structural DSGE model parameters based on the diagonal weighting matrix tend to be more accurate than intervals based on the optimal weighting matrix. They also appear more robust to the choice of the VAR lag order and to the maximum horizon of the impulse response function. Satisfactory coverage accuracy, however, may require fairly large sample sizes, even when using the diagonal weighting matrix. The coverage deficiencies in small samples can be traced to approximation error in the VAR representation of the DSGE model. When bootstrapping the state-space representation of the DSGE model directly rather than its VAR approximation, high coverage accuracy is obtained even in small samples, albeit at the cost of taking a stand on the parametric structure of the data generating process. These baseline simulation results pertain to strongly identified structural DSGE model parameters. We also provide simulation evidence that VAR-based bootstrap confidence intervals that allow for weak identification tend to be reasonably accurate even in realistically small samples and are not systematically less accurate than their DSGE model bootstrap counterparts.

In Section 4, we illustrate the implementation of the proposed methods in the context of a prototypical medium-scale New Keynesian DSGE model of the type used at many central banks. This empirical example illustrates that basing estimates of the asymptotic standard error on the standard deviation of the quasi-posterior of the structural parameters results in much lower standard error estimates than the alternative estimation methods developed in this paper. For example, whereas the point estimate of the price-markup factor is quite robust to the choice of method, its standard error is about three times as large one would have concluded based on the standard deviation of the quasi-posterior. These results are based on the conventional premise in empirical work that the structural parameters are strongly identified. We also present alternative estimates that take account of the possibility that some parameters are only weakly identified. We illustrate that allowing for weak identification in some cases affects the substantive conclusions, while in others it does not. The concluding remarks are in Section 5. The proofs are in Appendix A and Appendix B.

2. Asymptotic theory

The thought experiment is that the data are generated by a DSGE model. At least some of the structural parameters of this DSGE model are unknown. The DSGE model is approximated by a finite-order structural VAR model with identifying restrictions that are consistent with the underlying DSGE model.[1] The objective is to recover an estimate of the unknown structural parameters in the DSGE model by searching the space of these parameters for the parameter values that result in the closest match between the structural VAR impulse responses based on the actual data and those from the DSGE model evaluated at the hypothesized parameter values. We are concerned with the asymptotic properties of this impulse response matching estimator in repeated sampling. As is standard in this literature, it is assumed that the structural impulse responses obtained from the VAR model are strongly identified.

1 Fernandez-Villaverde et al. (2007) make precise the conditions under which a DSGE model may be approximated by a finite-order VAR model.