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Estimating smooth structural change in cointegration models[☆]

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ABSTRACT

This paper studies nonlinear cointegration models in which the structural coefficients may evolve smoothly over time, and considers time-varying coefficient functions estimated by nonparametric kernel methods. It is shown that the usual asymptotic methods of kernel estimation completely break down in this setting when the functional coefficients are multivariate. The reason for this breakdown is a kernel-induced degeneracy in the weighted signal matrix associated with the nonstationary regressors, a new phenomenon in the kernel regression literature. Some new techniques are developed to address the degeneracy and resolve the asymptotics, using a path-dependent local coordinate transformation to re-orient coordinates and accommodate the degeneracy. The resulting asymptotic theory is fundamentally different from the existing kernel literature, giving two different limit distributions with different convergence rates in the different directions of the (functional) parameter space. Both rates are faster than the usual root-*nh* rate for nonlinear models with smoothly changing coefficients and local stationarity. In addition, local linear methods are used to reduce asymptotic bias and a fully modified kernel regression method is proposed to deal with the general endogenous nonstationary regressor case, which facilitates inference on the time varying functions. The finite sample properties of the methods and limit theory are explored in simulations. A brief empirical application to macroeconomic data shows that a linear cointegrating regression is rejected but finds support for alternative polynomial approximations for the time-varying coefficients in the regression.

1. Introduction

Cointegration models are now one of the most commonly used frameworks for applied research in econometrics, capturing long term relationships among trending macroeconomic time series and present value links between asset prices and fundamentals in finance. These models conveniently combine stochastic trends in individual series with linkages between series that eliminate trending behavior and reflect latent regularities in the data. In spite of their importance and extensive research on their properties (e.g. Park and Phillips, 1988; Johansen, 1988; Phillips, 1991; Saikkonen, 1995; among many others) linear cointegration models are often rejected by the data even when there is clear co-movement in the series.

Various nonlinear parametric cointegrating models have been suggested to overcome such deficiencies. These models have been the subject of an increasing amount of econometric research following the development of methods for handling nonlinear nonstationary process asymptotics (Park and Phillips, 1999, 2001). However, parameter instability and functional form misspecification may limit the performance of such nonlinear parametric cointegration models in empirical applications (Hong and Phillips, 2010; Kasparis and Phillips, 2012; Kasparis et al., 2014). Most

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recently, therefore, attention has been given to flexible nonparametric and semiparametric approaches that can cope with the unknown functional form of responses in a nonstationary time series setting (Karlsen et al., 2007; Wang and Phillips, 2009a,b, 2016; Gao and Phillips, 2013a). A further extension of the linear framework allows cointegrating relationships to evolve smoothly over time using time-varying cointegrating coefficients (e.g. Park and Hahn, 1999; Juhl and Xiao, 2005; Cai et al., 2009; Xiao, 2009). This framework seems particularly well suited to empirical applications where there may be structural evolution in a relationship over time, thereby tackling one of the main limitations of fixed coefficient linear and nonlinear formulations. It is this framework that is the subject of the present investigation.

More specifically, we consider the following cointegration model with time-varying coefficient functions

$$y_t = x_t' f\left(t/n\right) + u_t = x_t' f_t + u_t, \ \ t = 1, \ldots, n, \tag{1.1}$$

where $f(\cdot)$ is a d-dimensional function of time (measured as a fraction of the sample size), x_t is an $I(1)$ vector, and u_t is a scalar process. The function $f(t/n)$ is sometimes called a fixed design and, in the present context, may be regarded as a weak trend function so that the model (1.1) captures potential drifts in the cointegrating linkage relationship between y_t and x_t over time. Such a modeling structure is especially useful for time series data over long horizons where economic mechanisms are likely to evolve and be subjected to changing institutional or regulatory conditions. For example, firms may change production processes in response to technological innovation and consumers may change consumption and savings behavior in response to new products and new banking regulations. These changes may be captured by temporal evolution in the coefficients through the functional dependence $f(t/n)$ in the model (1.1). Thus model (1.1) allows the long term relationships among the trending time series to evolve smoothly over time, which provides a more flexible framework than the parametric linear and nonlinear cointegration models. Some recent papers including Cai et al. (2009), Xiao (2009), Gao and Phillips (2013b) and Li et al. (2017) studied a nonlinear cointegrating model with functional coefficients and its generalized version, where the index variable in the functional coefficients is random, and developed the associated asymptotic theory. However, it is often difficult to select an appropriate random covariate as the index variable in practical applications and the requisite data may not be available. Such considerations partly motivate the use of a generic time-varying function to explore potential evolution in the cointegrating relationship between y_t and x_t in model (1.1). Nonparametric inference about time-varying parameters has received attention for modeling stationary or locally stationary time series data—see, for instance, Robinson (1989), Cai (2007), Li et al. (2011), Chen and Hong (2012), and Zhang and Wu (2012). However, there is little literature on this topic for integrated or cointegrated time series. One exception is Park and Hahn (1999), who considered the time-varying parameter model (1.1) and used sieve methods to transform the nonlinear cointegrating equation to a linear approximation with a sieve basis of possibly diverging dimension. Their asymptotic theory can be seen as an extension of the work by Park and Phillips (1988).

The present paper seeks to uncover evolution in the modeling framework for nonstationary time series over a long time horizon by using nonparametric kernel regression methods to estimate $f(\cdot)$, and our asymptotic theory is fundamentally different from that in the paper by Park and Hahn (1999). Our treatment shows that estimation of this model by conventional kernel methods encounters a degeneracy problem in the weighted signal matrix (the denominator of the kernel estimator (2.1)), which introduces a major new challenge in developing the limit theory. In fact, kernel degeneracy of this type can arise in many contexts where multivariate time-varying functions are associated with nonstationary regressors. The present literature appears to have overlooked the problem and existing mathematical tools fail to address it. The reason for degeneracy in the limiting weighted signal matrix is that kernel regression concentrates attention on a particular (time) coordinate, thereby fixing attention on a particular coordinate of f and the associated limit process of the regressor. In the multivariate case this focus on a single time coordinate produces a limiting signal matrix of deficient rank one whose zero eigenspace depends on the value of the limit process at that time coordinate. In other words, kernel degeneracy in the signal matrix is random and trajectory dependent.

This paper introduces a novel method to accommodate the degeneracy in kernel limit theory. The method transforms coordinates to separate the directions of degeneracy and non-degeneracy and proceeds to establish the kernel limit theory in each of these directions. The asymptotics are fundamentally different from those in the existing literature. As intimated, the transformation is path dependent and local to the coordinate of concentration. Two different convergence rates are obtained for different directions (or combinations) of the multivariate nonparametric estimators, and both of the two rates are faster than the usual ($\sqrt{nh}$) rate of stationary kernel asymptotics. Thus, two types of super-consistency exist for the nonparametric kernel estimation of time-varying coefficient functions, which we refer to as type I and type II super-consistency. The higher rate of convergence ($n\sqrt{h}$) lies in the direction of the nonstationary regressor vector at the local coordinate point and exceeds the usual $\sqrt{nh}$-rate by $\sqrt{n}$ (type I super-consistency). The lower rate (nh) lies in the degenerate direction but is still super-consistent (type II super-consistency) for nonparametric estimators and exceeds the usual $\sqrt{nh}$-rate by $\sqrt{nh}$.

The above results are all obtained for the Nadaraya–Watson local level time varying coefficient regression in a cointegrating model. Similar results are shown to apply for local linear time-varying regression which assists in reducing asymptotic bias. The general case of endogenous cointegrating regression is also included in our framework and a fully modified (FM; (Phillips and Hansen, 1990)) kernel method is proposed to address the endogeneity of the nonstationary regressors. In the use of this method it is interesting to discover that the kernel estimators need to be modified through bias correction only in the degenerate direction as the limit distribution of the estimators is not affected by the possible endogeneity in the direction of the nonstationary regressor vector at the local coordinate point. The limit theory for FM kernel regression also requires new asymptotic results on the consistent estimation of long run covariance matrices, which in turn involve uniform consistency arguments because of the presence of nonparametric regression residuals in these estimates. Importantly, inference about the time varying coefficient functions is unaffected by the degeneracy once the FM correction is made.

The remainder of the paper is organized as follows. Estimation methodology, some technicalities, and assumptions are given in Section 2. This section also introduces the kernel degeneracy problem, explains the phenomenon, and provides intuition for its resolution. Asymptotic properties of the nonparametric kernel estimator are developed in Section 3 with accompanying discussion. A kernel weighted FM regression method is proposed with attendant limit theory in Section 4. Section 5 reports simulation findings on the finite sample properties of the methods and limit theory, and gives a practical application of these time-varying kernel regression methods to empirical relationships involving consumption, disposable income, investment and real interest rates. Section 6 concludes the paper. Proofs of the main theoretical results in the paper are given in Appendix A. Some supplementary technical materials and discussions on model specification testing are provided in an Online Supplement (Phillips et al., 2016).