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Semiparametric dynamic portfolio choice with multiple conditioning variables☆

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ABSTRACT

Dynamic portfolio choice has been a central and essential objective for investors in active asset management. In this paper, we study the dynamic portfolio choice with multiple conditioning variables, where the dimension of the conditioning variables can be either fixed or diverging to infinity at certain polynomial rate of the sample size. We propose a novel data-driven method to estimate the optimal portfolio choice, motivated by the model averaging marginal regression approach suggested by Li et al. (2015). More specifically, in order to avoid the curse of dimensionality associated with the multivariate nonparametric regression problem and to make it practically implementable, we first estimate the marginal optimal portfolio choice by maximizing the conditional utility function for each univariate conditioning variable, and then construct the joint dynamic optimal portfolio through the weighted average of the marginal optimal portfolio across all the conditioning variables. Under some regularity conditions, we establish the large sample properties for the developed portfolio choice procedure. Both the simulation study and empirical application well demonstrate the finite-sample performance of the proposed methodology.

1. Introduction

Portfolio choice is a central issue for investors and asset managers. Financial research has clarified how this might be carried out to meet various objectives. Fundamental contributions to this literature have been made, inter alia, by: Markowitz (1952), Sharpe (1963), Merton (1969), Samuelson (1969), and Fama (1970). See Back (2010) and Brandt (2010) for some recent surveys. In practice, it is not uncommon that dynamic portfolio choice depends on many conditioning (or forecasting) variables, which reflect the varying investment opportunities over time. Generally speaking, there are two ways to characterize the dependence of portfolio choice on the conditioning variables. One is to assume a parametric statistical model that relates the returns of risky assets to the conditioning variables and then solve for an investor's portfolio choice by using some traditional econometric approaches to estimate the conditional distribution of the returns. However, the pre-assumed parametric model might be misspecified, which would lead to inconsistent or biased estimation of the optimal portfolio. The other way, which avoids the possible issue of model misspecification, is to use some nonparametric methods such as the kernel estimation method to characterize the dependence of the portfolio choice on conditioning variables. This latter method is first introduced by Brandt (1999), who also establishes the asymptotic properties for the estimated portfolio choice and provides an empirical application.

Although the nonparametric method allows the financial data to "speak for themselves" and is robust to model misspecification, its performance is often poor when the dimension of the conditioning variables is large (say, larger than three), owing to the so-called "curse of dimensionality" (c.f. Fan and Yao, 2003). This indicates that a direct use of Brandt's (1999) nonparametric method may be inappropriate when there are multiple conditioning variables. Our main objective in this paper is to address this issue in dynamic portfolio choice problem with multiple conditioning variables and propose a novel data-driven method to estimate the optimal portfolio choice, where the dimension of the conditioning variables and the number of the risky assets can be either fixed or diverging to infinity at certain polynomial rate of the sample size.

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In order to avoid the curse of dimensionality issue, we first consider the optimal portfolio choice for a given univariate conditioning variable, and then construct the joint dynamic optimal portfolio choice through a weighted average of the marginal optimal portfolio across all the conditioning variables. This method is partly motivated by the Model Averaging MArginal Regression (MAMAR) approach suggested in a recent paper by Li et al. (2015), which shows that such a method performs well in estimating the conditional multivariate mean regression function and out-of-sample prediction. Furthermore, we introduce a semiparametric data-driven method to choose the optimal weights in model averaging. Under some mild conditions, we establish the large sample properties for the developed portfolio choice procedure to show its advantages over the conventional nonparametric kernel smoothing method in terms of convergence. Both simulation studies and an empirical application are carried out to examine the finite sample performance of the proposed methodology.

The structure of the paper is as follows. The methodology for estimating the dynamic portfolio choice is introduced in Section 2, and the relevant large sample theory is presented in Section 3. The data-driven choice of the optimal weights in model averaging of the marginal optimal portfolios across all conditioning variables is developed in Section 4. Numerical studies including both simulation and an empirical application are reported in Section 5. Section 6 concludes the paper. The assumptions and the technical proofs of the main results are relegated to Appendices A and B, respectively.

2. Methodology for estimating dynamic portfolio choice

Suppose that there are n risky assets with $\mathbf{R}_t = (R_{1t}, \ldots, R_{nt})^\intercal$ as a vector of gross returns at time t, $t = 1, \ldots, T$, where n can be either fixed or diverging to infinity with the sample size T. Let $\mathbf{X}_t = (X_{1t}, \ldots, X_{Jt})^\intercal$, where J is the number of conditioning or forecasting variables X_{jt}. The dynamic portfolio choice aims to choose the portfolio weights at each time period t by maximizing the conditional utility function defined by

$$\mathsf{E}\left[u(w^\intercal \mathbf{R}_t)|\mathbf{X}_{t-1}\right] = \mathsf{E}\left[u(w^\intercal \mathbf{R}_t)|(X_{1,t-1}, \ldots, X_{J,t-1})\right], \tag{2.1}$$

subject to $\mathbf{1}_n^\intercal w = \sum_{i=1}^n w_i = 1$, where $w = (w_1, \ldots, w_n)^\intercal$, $\mathbf{1}_n$ is the n-dimensional column vector of ones, $u(\cdot)$ is a concave utility function which measures the investor's utility with wealth $w^\intercal \mathbf{R}_t$ at time t. For simplicity, we only focus on the problem of single-period portfolio choice. Furthermore, we assume that the investors can borrow assets and sell them (short selling), which indicates that some of the portfolio weights may take negative values.

The classic mean–variance paradigm considers the quadratic utility function $u(v) = v - (\gamma/2)v^2$ or the CARA (Constant Absolute Risk Aversion) utility function $u(v) = -\exp(-\gamma v)$ plus normality, in which case the solution (with covariates) is explicitly defined in terms of the conditional mean vector $\boldsymbol{\mu}(\mathbf{x}) = \mathsf{E}[\mathbf{R}_t|\mathbf{X}_{t-1} = \mathbf{x}]$, $\mathbf{x} = (x_1, \ldots, x_J)^\intercal$, and the conditional covariance matrix $\boldsymbol{\Sigma}(\mathbf{x}) = \mathsf{E}[(\mathbf{R_t} - \boldsymbol{\mu}(\mathbf{x}))(\mathbf{R_t} - \boldsymbol{\mu}(\mathbf{x}))^\intercal|\mathbf{X}_{t-1} = \mathbf{x}]$ of returns, i.e.,

$$w(\mathbf{x}) = \frac{1}{\gamma}\boldsymbol{\Sigma}^{-1}(\mathbf{x})\left[\boldsymbol{\mu}(\mathbf{x}) - \theta(\mathbf{x})\mathbf{1}_n\right],$$

$$\theta(\mathbf{x}) = \frac{\boldsymbol{\mu}(\mathbf{x})^\intercal \boldsymbol{\Sigma}^{-1}(\mathbf{x})\mathbf{1}_n - \gamma}{\mathbf{1}_n^\intercal \boldsymbol{\Sigma}^{-1}(\mathbf{x})\mathbf{1}_n}.$$

In this case, it suffices to know $\boldsymbol{\mu}(\cdot)$ and $\boldsymbol{\Sigma}(\cdot)$. One may also work with the more general CRRA (Constant Relative Risk Aversion) utility function with risk aversion parameter γ

$$u(v) = \begin{cases} \dfrac{v^{1-\gamma}}{1-\gamma}, & \gamma \neq 1 \\ \log v, & \gamma = 1, \end{cases}$$

in which case the solution for the optimal weights is not typically explicit, and generally depends on more features of the conditional distribution. More discussion on different classes of utility functions $u(\cdot)$ can be found in Chapter 1 of the book by Back (2010).

In order to solve the general maximization problem in (2.1), Brandt (1999) proposes a nonparametric conditional method of moments approach, which can be seen as an extension of the method of moments approach in Hansen and Singleton (1982). Taking the first-order derivative of $u(\cdot)$ in (2.1) with respect to w_i and considering the constraint of $\mathbf{1}_n^\intercal w = \sum_{i=1}^n w_i = 1$, we may obtain the dynamic portfolio choice by solving the following equations for $w_1, \ldots, w_{n-1}$:

$$\mathsf{E}\left[(R_{it} - R_{nt})\dot{u}(w^\intercal \mathbf{R}_t)|X_{1,t-1}, \ldots, X_{J,t-1}\right] = 0 \quad \text{a.s.}, i = 1, \ldots, n-1, \tag{2.2}$$

where $\dot{u}(\cdot)$ is the derivative of the utility function $u(\cdot)$. The last element w_n in w can be determined by using the constraint $\sum_{i=1}^n w_i = 1$. Brandt (1999) suggests a kernel-based smoothing method to estimate the solution to (2.2). However, when J is large, the kernel-based nonparametric conditional method of moments approach would perform quite poorly due to the curse of dimensionality discussed in Section 1. Therefore, we propose a novel dimension-reduction technique to address this problem.

We start with the portfolio choice for each univariate conditioning variable in $\mathbf{X}_{t-1}$. For $j = 1, \ldots, J$, we define the marginal conditional utility function as

$$\mathsf{E}\left[u(w^\intercal \mathbf{R}_t)|X_{j,t-1} = x_j\right] \tag{2.3}$$

with the constraint $\mathbf{1}_n^\intercal w = \sum_{i=1}^n w_i = 1$. The associated first-order conditions for the marginal optimal portfolio weights $w_j(x_j)$ evaluated at x_j for the conditioning variables are:

$$\mathsf{E}\left[(R_{it} - R_{nt})\dot{u}(w_j^\intercal(x_j)\mathbf{R}_t)|X_{j,t-1} = x_j\right] = 0, \quad i = 1, \ldots, n-1, \tag{2.4}$$

where

$$w_j(x_j) = [w_{1j}(x_j), \ldots, w_{nj}(x_j)]^\intercal \quad \text{with } w_{nj}(x_j) = 1 - \sum_{i=1}^{n-1} w_{ij}(x_j).$$

For a given j, this is essentially the problem posed and solved by Brandt (1999). For given $\mathbf{x} = (x_1, \ldots, x_J)^\intercal$, (2.3) and (2.4) may be understood as the utility function and the corresponding first order conditions for portfolio choice in a "fictitious economy", where the realization of each univariate conditioning variable determines the state of the economy.

We next consider how to combine the marginal portfolios selected above to form a joint portfolio. We shall consider a weighted average of the marginal portfolio choices $w_j(x_j)$ over $j = 1, \ldots, J$, and obtain the joint portfolio choice as

$$w_{\mathbf{a}}(\mathbf{x}) = \sum_{j=1}^J a_j w_j(x_j) \quad \text{with } \sum_{j=1}^J a_j = 1, \tag{2.5}$$

where negative values for a_j can be allowed. In Section 4, we will discuss how to choose the weights $\mathbf{a} = (a_1, \ldots, a_J)^\intercal$ in the combination (2.5) by using a data-driven method.

The joint portfolio choice $w_{\mathbf{a}}(\mathbf{x})$ defined in (2.5) can, in some sense, be seen as an *approximation* of the true optimal portfolio choice, as we next discuss. Consider the following class of weights (that are measurable functions of the covariates):

$$\mathcal{W} = \left\{w(\cdot) : \sum_{i=1}^n w_i(\mathbf{x}) = 1\right\}$$

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