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Modeling covariance breakdowns in multivariate GARCH[☆]

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ABSTRACT

This paper proposes a flexible way of modeling dynamic heterogeneous covariance breakdowns in multivariate GARCH models through a stochastic component that allows for changes in the conditional variances, covariances and implied correlation coefficients. Different breakdown periods will have different impacts on the conditional covariance matrix and are estimated from the data. We propose an efficient Bayesian posterior sampling procedure and show how to compute the marginal likelihood. Applied to daily stock market and bond market data, we identify a number of different covariance breakdowns which leads to a significant improvement in the marginal likelihood and gains in portfolio choice.

1. Introduction

This paper proposes a flexible way of modeling dynamic heterogeneous covariance breakdowns in multivariate GARCH (MGARCH) models. During periods of normal market activity, volatility dynamics are governed by an MGARCH specification. A covariance breakdown is any significant temporary deviation of the conditional covariance matrix from its implied MGARCH dynamics. A covariance breakdown is captured through a flexible stochastic component that allows for changes in the conditional variances, covariances and implied correlation coefficients.

It is widely acknowledged that markets face periods that are characterized by abnormal behavior. Several approaches have been used to capture changes in the dynamics of conditional second moments including dynamic copulas (Kenourgios et al., 2011; Christoffersen et al., 2012), and the factor spline GARCH model of Rangel and Engle (2012).[1] Dufays (2013) uses an infinite-state hidden Markov model to allow for parameter change in Engle's (2002) dynamic conditional correlation model. The path dependence that the latent state variable causes in the GARCH recursions is removed following the ideas in Klaassen (2002).[2] Haas and Mittnik (2008) and Chen (2009) extend the univariate MS-GARCH model in Haas et al. (2004) to a multivariate setting. Their model assumes there are K parallel MGARCH models running at the same time, where K is the number of states. Silvennoinen and Teräsvirta (2009) apply the smooth transition modeling approach to conditional correlations. Other regime-switching approaches include Ang and Bekaert (2004), Guidolin and Timmermann (2006) and Pelletier (2006).

In contrast to the literature, which has tended to focus on correlation breakdowns, we investigate breakdowns in each

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[1] It is important to account for changes in GARCH dynamics. For instance, in the univariate setting, neglected parameter changes in volatility dynamics can bias GARCH parameter estimates toward higher persistence and lead to poor forecasts of volatility (Lamoureux and Lestrapes, 1990; Hillebrand, 2005).

[2] Another approach to dealing with path dependence directly is the particle MCMC approach of Bauwens et al. (2014).

component of the conditional covariance matrix. This has several advantages. First, we can see how conditional correlations are affected through variances and covariances. Second, by modeling the full covariance matrix we avoid issues of misspecification by focusing only on correlations (Forbes and Rigobon, 2002) and neglecting heteroskedasticity. In our model a covariance breakdown does not necessarily imply a correlation breakdown or contagion effect. It depends on the relative changes in the conditional covariance and conditional variances. Empirically we identify both covariance breakdowns which lead to correlation changes and breakdowns which have little impact on correlations.

To our knowledge this is the first paper to explicitly model the dynamics of conditional covariance breakdowns and estimate their impacts. In our approach a covariance breakdown is any sustained deviation of the conditional second moments from the covariance matrix implied by the MGARCH specification. Each breakdown period is different and estimated from the data. Covariance breakdowns as well as normal periods are assumed to follow a first-order Markov chain. Each breakdown is characterized by a random matrix drawn from an inverse-Wishart distribution that scales (multiplies) the MGARCH covariance matrix.[3] This stochastic scale matrix can change several times over the course of a single breakdown. This approach is very flexible and allows a single breakdown to display different characteristics over time while retaining a positive definite matrix. Since covariance breakdowns are finite, they eventually end and we return to a model in which the MGARCH dynamics solely determine conditional second moments.

Our model can be considered as an extension to Markov switching models. Bayesian inference for Markov regime-switching models is usually carried out based on the forward-backward algorithm of Chib (1996). Our approach is different than the conventional regime-switching specification in which model parameters governing a time period are selected from a fixed parameter set. A covariance breakdown is captured by introducing an exogenous stochastic multiplicative component to the volatility matrix itself. This requires a new posterior sampling approach for the states. We construct an efficient sampling scheme to sample the unobserved state variables as well as other fixed parameters.

Whether covariance breakdowns are supported by the data can be formally assessed in the context of Bayesian model comparison by making use of the marginal likelihood. We show how to compute the marginal likelihood and design a particle filter for the task.

The model is applied to daily excess returns on the S&P 500 index and short-term and long-term bonds over a twenty-five year period. Including fat-tailed return innovations in the model is important in distinguishing between outliers and sustained covariance breakdowns. We compare our model to an MGARCH model with Student-t innovations but with no breakdowns as well as a version of that model subject to Markov switching. Bayes factors strongly support the inclusion of covariance breakdowns. The volatility dynamics during breakdown periods are very different for the models as well as breakdowns being different over time. For example, in the recent financial crisis we identify an initial breakdown which leads to an overall increase in variability. This features large increases in conditional variances and drops in covariances between the stock and bond market. However, the conditional correlations do not show a dramatic change. Following this episode is another breakdown which is characterized as a reduction in overall variability.

Estimates indicate that covariance breakdowns occur 42% of the time and their expected duration is 1.5 months in our sample. The impact of a typical covariance breakdown is expected to increase variability. In addition to improving the fit of the data, modeling covariance breakdowns provides improved portfolio choice.

The rest of the paper is organized as follows. In Section 2, we introduce the breakdown model and discuss its properties. Section 3 constructs a sampling procedure for the posterior inference of the model. Section 4 provides simulation study for illustration. Section 5 shows how to compute the marginal likelihood of our model. In Section 6, we apply the model to study the volatility dynamics among the stock market and the bond market and Section 7 concludes. The Appendix contains details on posterior sampling and computation of the marginal likelihood.

2. Multivariate GARCH with covariance breakdowns

Consider a k-dimensional vector time series y_t, $t = 1, 2, 3, \ldots$. Let $\mathcal{F}_{t-1}$ be the sigma field generated by the past values of y_t until time $t-1$. Consider the following model

$$y_t = \mu + H_t^{1/2} \Lambda_t^{1/2} z_t, \tag{1}$$

where $\mu = \mathbb{E}(y_t|\mathcal{F}_{t-1})$ is the constant conditional mean[4] vector and $z_t \sim NID(0, I)$.[5] $H_t^{1/2}$ denotes the Cholesky factor of the $k \times k$ positive definite matrix H_t, which is assumed to follow any of the popular specifications for the MGARCH model. Popular examples of MGARCH models include, among others, the vector-diagonal GARCH (VDGARCH) by Ding and Engle (2001) and the dynamic conditional correlation (DCC) by Engle (2002). See Bauwens et al. (2006) for a review.

The dynamics of Λ_t depend on a latent discrete state variable $s_t \in \{1, 2, 3\}$. s_t follows a Markov chain whose transition matrix is represented as

$$P = \begin{pmatrix} \pi_1 & 1-\pi_1 & 0 \\ (1-\pi_2)\pi_4 & \pi_2 & (1-\pi_2)(1-\pi_4) \\ (1-\pi_3)\pi_5 & (1-\pi_3)(1-\pi_5) & \pi_3 \end{pmatrix}, \tag{2}$$

with each $\pi_i \in [0, 1]$, $i = 1, \ldots, 5$ being a free parameter. According to (2), moving directly from state 1 to state 3 is prohibited but all other moves are possible. While in state 2, the probability of staying is π_2; conditional on leaving, the probability of moving into state 1 is π_4. Similarly, while in state 3, the probability of staying is π_3; conditional on leaving, the probability of moving into state 1 is π_5 and the probability of moving to state 2 is $(1-\pi_5)$.

Let $s_{1:t} = \{s_1, \ldots, s_t\}$. Λ_t is then determined as follows

$$\Lambda_t|s_{1:t} = \begin{cases} I & \text{if } s_t = 1 \\ \Lambda_{t-1} & \text{if } s_t = s_{t-1} \\ \sim G_0 & \text{if } s_t \neq s_{t-1} \text{ and } s_t \neq 1. \end{cases} \tag{3}$$

Thus, when $s_t = 1$, Λ_t is the identity matrix. If s_t does not change, neither does Λ_t. Whenever s_t switches into 2 or 3, Λ_t is a new stochastic draw from G_0.

s_t divides the sample path into periods of *normal* states ($s_t = 1$) and periods of *covariance breakdown* states ($s_t = 2$ or $s_t = 3$). Switches out of state 1 and back into state 1 delineate a covariance breakdown. Therefore, a breakdown can be characterized by a sequence of states all equal to 2 with one associated Λ_t (1,2,2,2,2,1) or by alternating between states 2 and 3 (1,2,2,3,3,3,2,2,1) along with a different draw for Λ_t every time the state switches

[3] To be precise, a positive definite matrix drawn from an inverse-Wishart density is sandwiched between the Cholesky decomposition of the MGARCH matrix.

[4] A time-varying conditional mean can also be used.

[5] Other distributions such as a Student-t could be used for z_t as long as a normal decomposition can be admitted.